ICM ~Unit 3 ~ Day 3

Section 1.2— Practice with Horizontal Asymptotes, Domain and Discontinuities;

Introduction to Limits of functions;

End Behavior of graphs and Limits

Warm Up

1. Find the domain, then convert to fractional/rational exponent.

$$f(x) = \sqrt{7-x}$$

For #2 and 3, find the domain, x & y intercepts, and label any discontinuities (including if they are removable or non-removable).

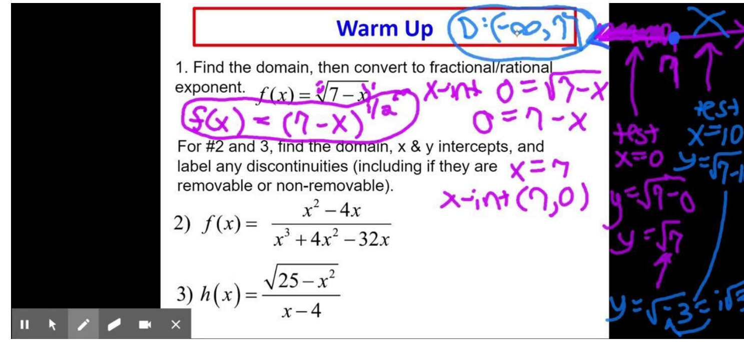
2) $$f(x) = \frac{x^2 - 4x}{x^3 + 4x^2 - 32x}$$

3) $$h(x) = \frac{\sqrt{25 - x^2}}{x - 4}$$

Warm Up

1. Find the domain, then convert to fractional/rational exponent.

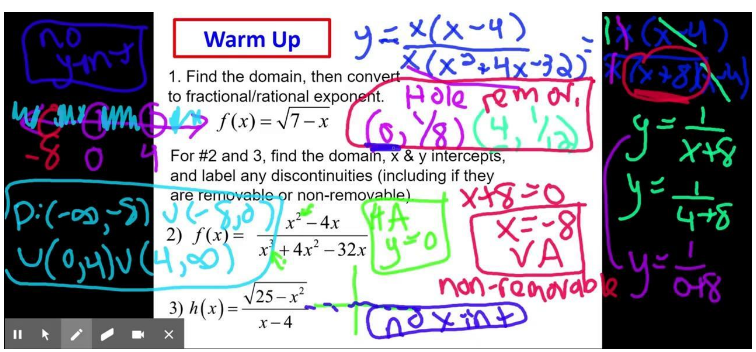
$$f(x) = \sqrt{7-x}$$

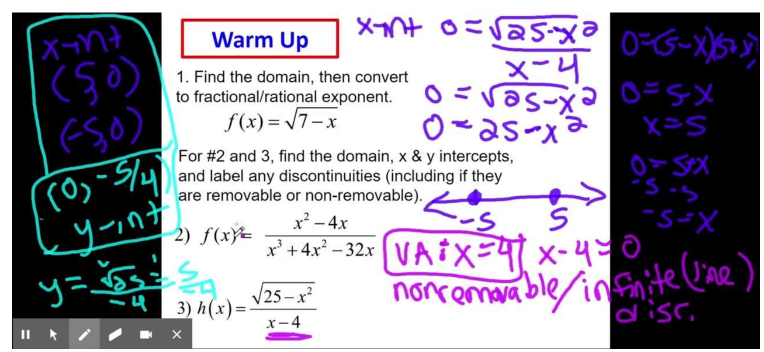


For #2 and 3, find the domain, x & y intercepts, and label any discontinuities (including if they are removable or non-removable).

$$2)\ \ f(x) = \frac{x^2-4x}{x^3+4x^2-32x}$$

$$3)\ h(x) = \frac{\sqrt{25-x^2}}{x-4}$$





Warm Up ANSWERS

1. Find the domain, then convert to fractional/rational exponent.

$$f(x)=\sqrt{7-x} = (7-x)^{\frac{1}{2}}$$

$$Domain: (-\infty, 7]$$

For #2 and 3, find the domain, x & y intercepts, and label any discontinuities (including if they are removable or non-removable).

2) $$f(x)=\frac{x^2-4x}{x^3+4x^2-32x}$$

$x-\text{int}: \textit{none}$

$y-\text{int}: \textit{none (hole there)}$

$$Hole\ (removable\ disc.): \ (0,\frac{1}{8})\ and\ (4,\frac{1}{12})$$

$$V.A.\ (non-removable\ disc.)\ x=-8$$

Warm Up ANSWERS (continued)

For #2 and 3, find the domain, x & y intercepts, and label any discontinuities (including if they are removable or non-removable).

$$3)\ h(x)=\frac{\sqrt{25-x^2}}{x-4}$$

$Domain: [-5,4)\cup(4,5]$

$x-\text{int}: (-5,\ 0) \,\&\, (5,0)$

$y-\text{int}: (0,-\frac{5}{4})$

Nonremovable Discontinuity (Vertical Asymptote) at x = 4

No Holes (No removable Discontinuities) because there is no common factor between the top and bottom of the fraction

Homework Questions?

Remember to check your HW answers on the website and bring questions to class meetings 😊

Tonight's Homework

- Packet p. 6 and
- 1st two pages of Rationals Summary & Practice Handout

Before notes,
a Rational Summary

Let's Summarize some rules and steps for discontinuities
– especially for Rational Functions…

Finding Discontinuities Summary

- To find vertical asymptotes and holes, factor the problem and simplify.
  - **If the factor in the bottom cancels out, it gives us a hole.**
    - **Removable** $f(x) = \frac{x+2}{(x+2)(x-1)}$

So the hole occurs at x = -2

$(-2, -\frac{1}{3})$

How do you find the y-value of the hole?

Simplify and substitute the x-value into the **remaining** equation!

$$f(-2) = \frac{1}{-2-1}$$

  - **If the factor does not cancel out, it gives us a vertical asymptote.**
    - **Nonremovable**

The vertical asymptote for f(x) is at x = 1

Rationals and Radicals Summary

Domain:

Consider the **vertical asymptotes** and the x-value of the **hole**

Make sure values under any radicals are positive

x-intercept: Best to find AFTER getting VA & hole

Set **y = 0** and solve for **x**.

y-intercept:

Set **x = 0** and solve for **y**.

Summary: Definition of Degree

- **Degree of a polynomial in one variable:** the value of the greatest exponent

Ex: $f(x) = 4x^2 + 9x + 8$ **Degree: 2**

Ex: $g(x) = -5x^3 + 6x^2 + 4x$ **Degree: 3**

Ex: $h(x) = 5x^2 + 3x^4 + 2x$ **Degree: 4**

Watch out…the polynomial may not be in order!!

- **Degree can help us with determining the horizontal asymptote of rational functions...**

Summary: Horizontal Asymptotes

For horizontal asymptotes, think BOSTON for ***polynomials***! Looking at the degree of top & bottom…

Bottom > Top $$f(x)=\frac{2x}{x^2+3x}$$ $H.A.: y=0$

y=**O**

Same = ratio $$g(x)=\frac{2x^3}{5x^3+4x^2}$$ $H.A.: \ y=\frac{2}{5}$

Top > Bottom

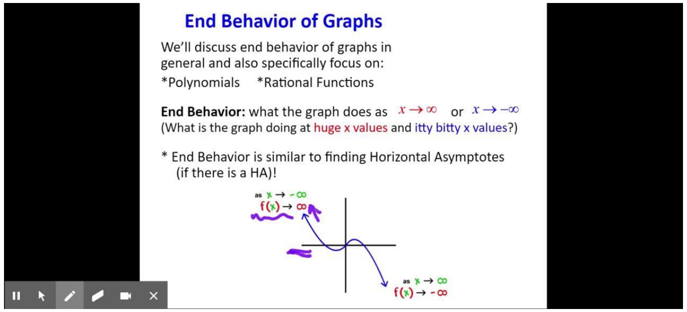
↑ **O** No HA. $$h(x)=\frac{5x^2}{7x+3}$$ $No\ H.A.$

N

Notes Day 3: Degree, Horizontal Asymptotes, and End Behavior

A Graphical Approach

End Behavior of Graphs

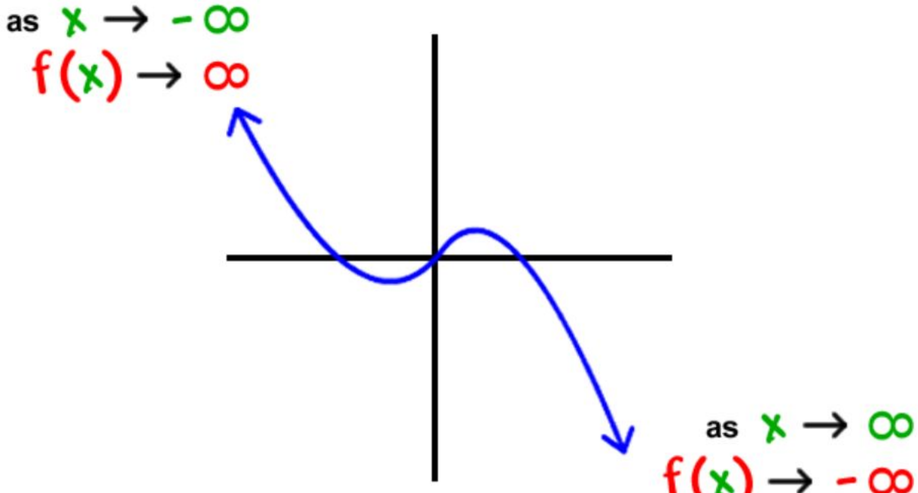


We'll discuss end behavior of graphs in general and also specifically focus on:

*Polynomials *Rational Functions

End Behavior: what the graph does as $x \to \infty$ or $x \to -\infty$
(What is the graph doing at huge x values and itty bitty x values?)

* End Behavior is similar to finding Horizontal Asymptotes (if there is a HA)!

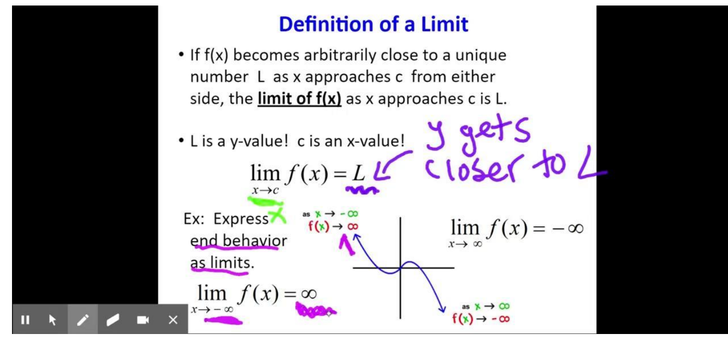


Definition of a Limit

- If f(x) becomes arbitrarily close to a unique number L as x approaches c from either side, the **limit of f(x)** as x approaches c is L.

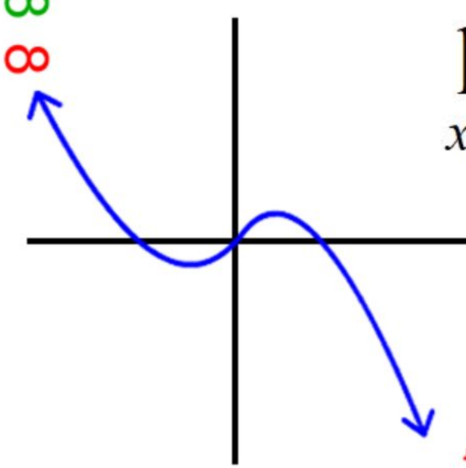
- L is a y-value! c is an x-value!

$$\lim_{x \to c} f(x) = L$$



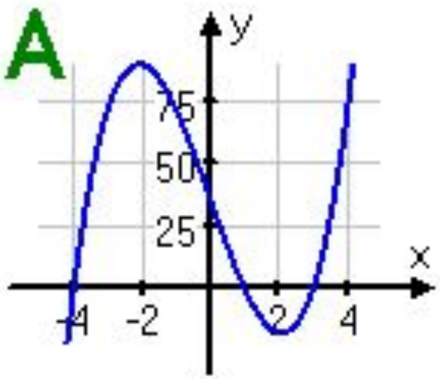
Ex: Express end behavior as limits.

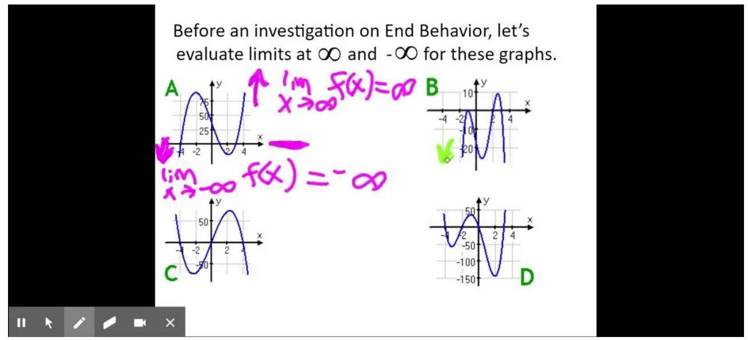
$$\lim_{x \to -\infty} f(x) = \infty$$

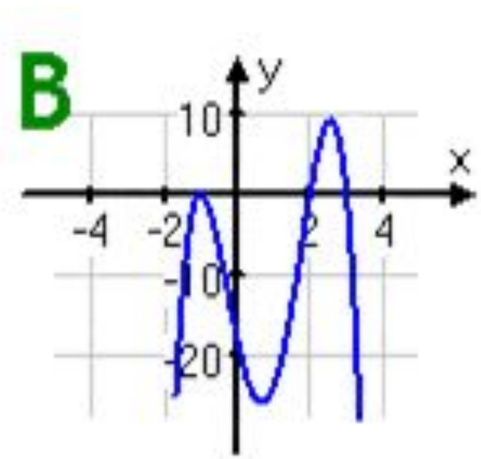


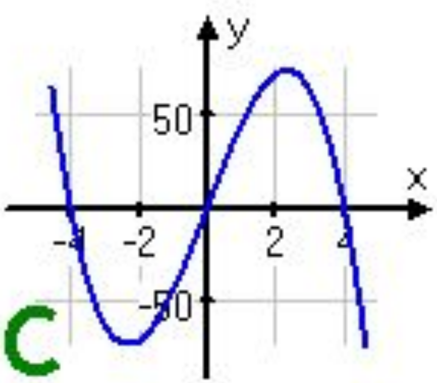
$$\lim_{x \to \infty} f(x) = -\infty$$

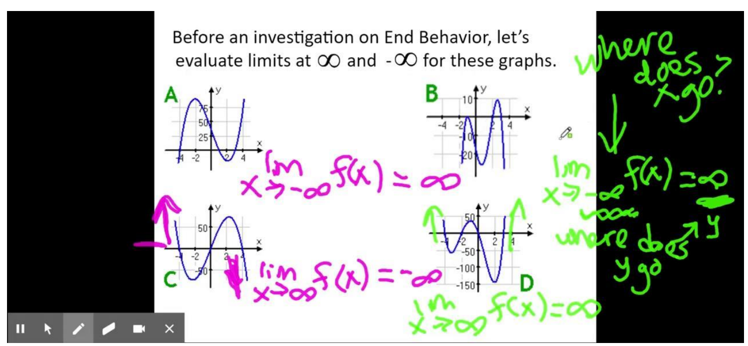
Before an investigation on End Behavior, let's evaluate limits at ∞ and -∞ for these graphs.

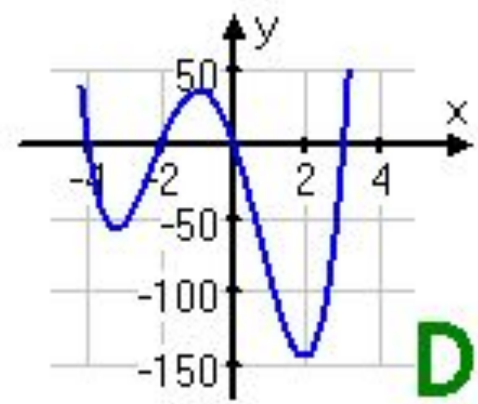






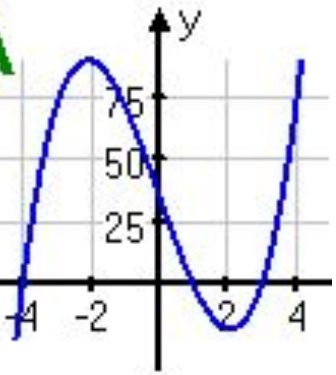






ANSWERS Before an investigation on End Behavior, let's evaluate limits at ∞ and $-\infty$ for these

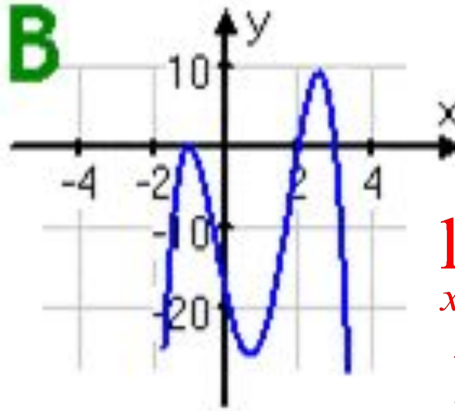
A



$$\lim_{x\to\infty} f(x) = \infty$$

$$\lim_{x\to-\infty} f(x) = -\infty$$

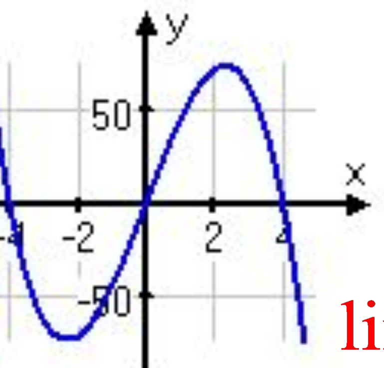
B



$$\lim_{x\to\infty} f(x) = -\infty$$

$$\lim_{x\to-\infty} f(x) = -\infty$$

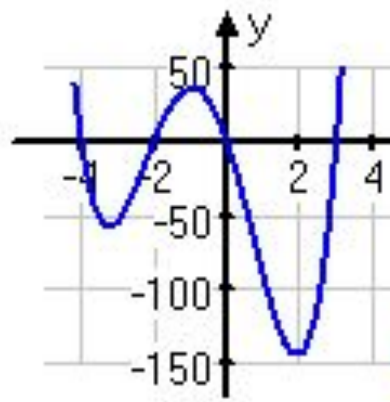
C



$$\lim_{x\to\infty} f(x) = -\infty$$

$$\lim_{x\to-\infty} f(x) = \infty$$

D

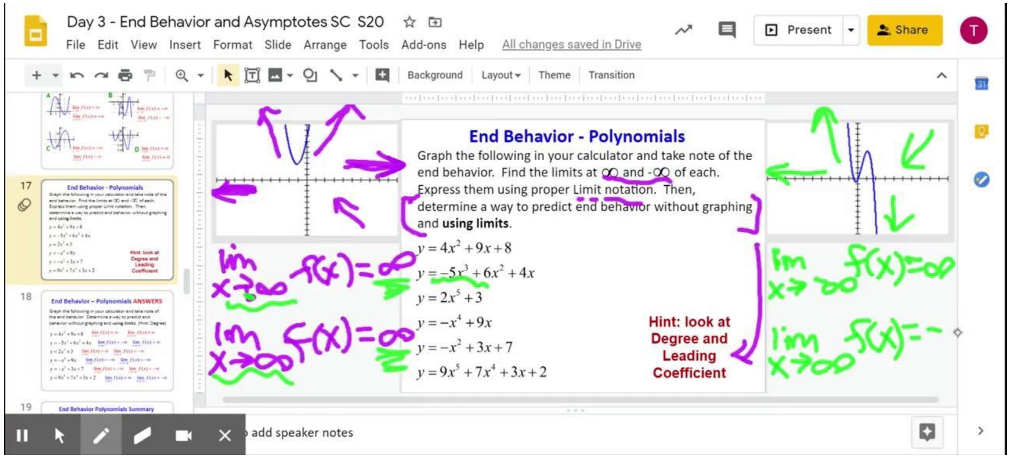


$$\lim_{x\to\infty} f(x) = \infty$$

$$\lim_{x\to-\infty} f(x) = \infty$$

End Behavior - Polynomials

Graph the following in your calculator and take note of the end behavior. Find the limits at ∞ and $-\infty$ of each. Express them using proper Limit notation. Then, determine a way to predict end behavior without graphing and **using limits**.



$$y = 4x^2 + 9x + 8$$

$$y = -5x^3 + 6x^2 + 4x$$

$$y = 2x^5 + 3$$

$$y = -x^4 + 9x$$

$$y = -x^2 + 3x + 7$$

$$y = 9x^5 + 7x^4 + 3x + 2$$

Hint: look at Degree and Leading Coefficient

End Behavior – Polynomials ANSWERS

Graph the following in your calculator and take note of the end behavior. Determine a way to predict end behavior without graphing and **using limits**. (Hint: Degree)

$y = 4x^2 + 9x + 8$ $\lim_{x \to \infty} f(x) = \infty$ $\lim_{x \to -\infty} f(x) = \infty$

$y = -5x^3 + 6x^2 + 4x$ $\lim_{x \to \infty} f(x) = -\infty$ $\lim_{x \to -\infty} f(x) = \infty$

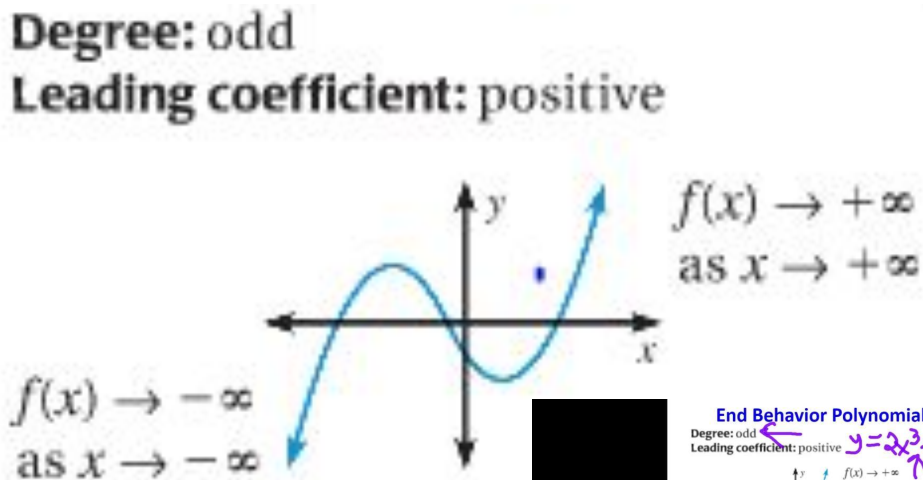
$y = 2x^5 + 3$ $\lim_{x \to \infty} f(x) = \infty$ $\lim_{x \to -\infty} f(x) = -\infty$

$y = -x^4 + 9x$ $\lim_{x \to \infty} f(x) = -\infty$ $\lim_{x \to -\infty} f(x) = -\infty$

$y = -x^2 + 3x + 7$ $\lim_{x \to \infty} f(x) = -\infty$ $\lim_{x \to -\infty} f(x) = -\infty$

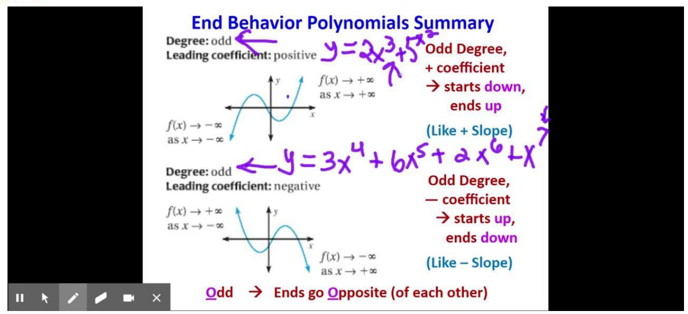
$y = 9x^5 + 7x^4 + 3x + 2$ $\lim_{x \to \infty} f(x) = \infty$ $\lim_{x \to -\infty} f(x) = -\infty$

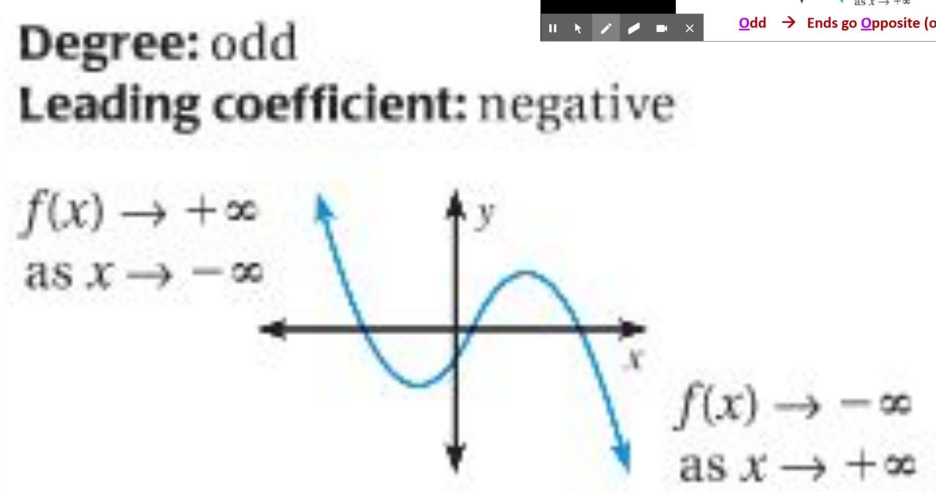
End Behavior Polynomials Summary



Odd Degree,
+ coefficient
□ starts down,
ends up

(Like + Slope)





Odd Degree,
— coefficient
□ starts up,
ends down

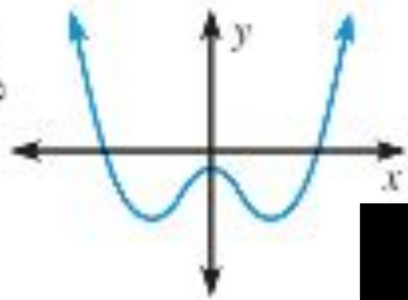
(Like – Slope)

Odd □ Ends go Opposite (of each other)

End Behavior Polynomials Summary

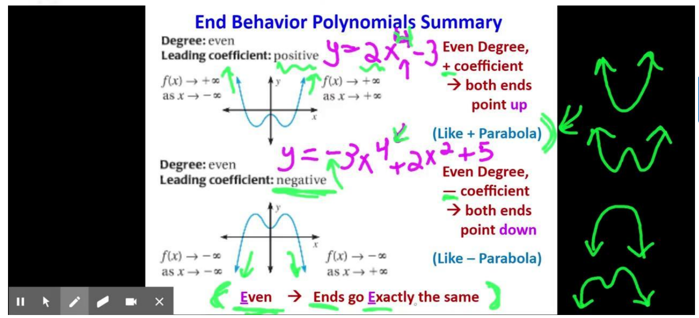
Degree: even
Leading coefficient: positive

$f(x) \to +\infty$ as $x \to -\infty$



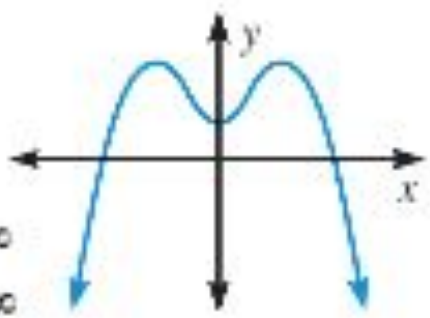
$f(x) \to +\infty$ as $x \to +\infty$

Even Degree,
+ coefficient
☐ both ends
point up



Like + Parabola)

Degree: even
Leading coefficient: negative



$f(x) \to -\infty$ as $x \to -\infty$

$f(x) \to -\infty$ as $x \to +\infty$

Even Degree,
— coefficient
☐ both ends
point down

(Like – Parabola)

Even ☐ Ends go Exactly the same

End Behavior: Rational Functions

- Rational functions: Ratio of two polynomials
- As you may have guessed, degrees of these two polynomials play a key role in determining the end behavior.

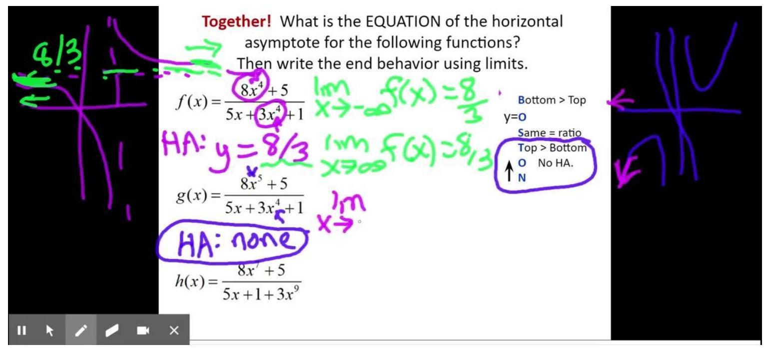
Consider the following scenarios:

1) The degree of the numerator is bigger than the degree of the denominator.
2) The degree of the numerator is the same as the degree of the denominator.
3) The degree of the numerator is smaller than the degree of the denominator.

Determine a way to predict end behavior of Rational Functions without graphing.

Together! What is the EQUATION of the horizontal asymptote for the following functions?
Then write the end behavior using limits.

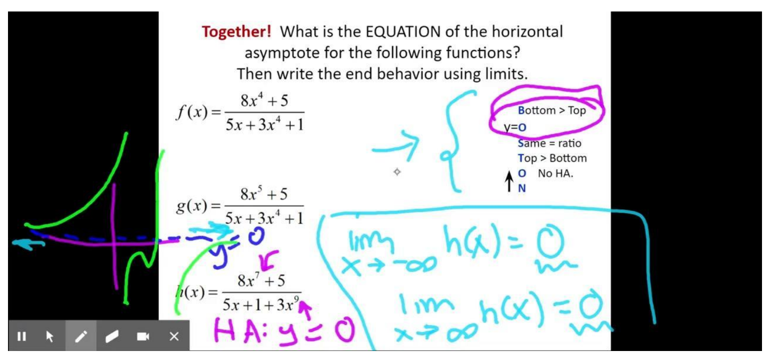
$$f(x)=\frac{8x^4+5}{5x+3x^4+1}$$



Bottom > Top
y=**O**
Same = ratio
Top > Bottom
↑ **O** No HA.
N

$$g(x)=\frac{8x^5+5}{5x+3x^4+1}$$

$$h(x)=\frac{8x^7+5}{5x+1+3x^9}$$



Together ANSWERS! What is the EQUATION of the horizontal asymptote for the following functions? Then write the end behavior using limits.

$$f(x) = \frac{8x^4 + 5}{5x + 3x^4 + 1}$$

$$H.A.: y = 8/3$$

$$\lim_{x \to -\infty} f(x) = \frac{8}{3}$$

$$\lim_{x \to \infty} f(x) = \frac{8}{3}$$

Bottom > Top
y=**O**
Same = ratio
Top > Bottom
O No HA.
N

$$g(x) = \frac{8x^5 + 5}{5x + 3x^4 + 1}$$

$$H.A.: none$$

$$\lim_{x \to \infty} g(x) = \infty$$

$$\lim_{x \to -\infty} g(x) = -\infty$$

$$h(x) = \frac{8x^7 + 5}{5x + 1 + 3x^9}$$

$$H.A.: y = 0$$

$$\lim_{x \to -\infty} f(x) = 0$$

$$\lim_{x \to \infty} f(x) = 0$$

Homework

- Packet p. 6 and
- 1st two pages of Rationals Summary & Practice Handout