NOTES Spr '18

Day 2

Limit Definition of Derivatives

The Algebra Behind Derivatives

Warm Up:

Given the function $f(x) = x^2 + 7$:

Find:

1. $f(x + 3) =$
2. $f(x + h) =$
3. $\lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$
4. $\frac{x}{3} + \frac{2x}{4}$
5. $\frac{3}{x} + \frac{4}{x-2}$

Warm Up:
Given the function $f(x) = x^2 + 7$:

Find:
1. $f(x + 3) =$ $\boxed{x^2+6x+16}$
2. $f(x + h) =$ $\boxed{x^2+2xh+h^2}$
3. $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ $= \boxed{2x}$

4. $\frac{4 \cdot x}{4 \cdot 3} + \frac{2x \cdot 3}{4 \cdot 3}$

$\frac{4x}{12} + \frac{6x}{12} = \frac{10x}{12}$

$\boxed{\frac{5x}{6}}$

5. $\frac{3}{x} + \frac{4}{x-2}$

$\frac{3(x-2)}{x(x-2)} + \frac{4(x)}{(x-2)(x)}$

$\frac{3x-6}{x(x-2)} + \frac{4x}{x(x-2)}$

$\boxed{\frac{7x-6}{x(x-2)}}$

Get common denom. 1st!!

PREREQUISITE REMINDERS

$\sqrt{x}\cdot\sqrt{x} = x$ $\quad (\sqrt{x})^2 = \sqrt{x^2} = x$ $\quad (x+3)(x-3) = x^2 - 3x + 3x - 9 = x^2 - 9$

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$\sqrt{x+h}\cdot\sqrt{x+h} = x+h$ $\quad (\sqrt{x+h})^2 = x+h$ $\quad (x+4)(x-4) = x^2 - 16$

$\sqrt{x+2}\cdot\sqrt{x+2} = x+2$ $\quad (\sqrt{x+2})^2 = x+2$ $\quad (\sqrt{x}+4)(\sqrt{x}-4) = x-16$

Example 3:

Evaluate the derivative using the limit definition of derivatives. $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

- Function: $f(x) = \sqrt{x-2}$

For Square Root problems, you must use the conjugate!

REMEMBER:

$\sqrt{a+4} + \sqrt{a}$ it's conjugate is $\sqrt{a+4} - \sqrt{a}$

** use conjugate

$$f'(x) = \lim_{h\to 0}\frac{\left(\sqrt{x+h-2} - \sqrt{x-2}\right)}{h}\cdot\frac{\left(\sqrt{x+h-2} + \sqrt{x-2}\right)}{\left(\sqrt{x+h-2} + \sqrt{x-2}\right)}$$

* square root and square undo eachother

$$= \lim_{h\to 0}\frac{(\sqrt{x+h-2})^2 + \cancel{\sqrt{x+h-2}\sqrt{x-2}} - \cancel{\sqrt{x-2}\sqrt{x+h-2}} - (\sqrt{x-2})^2}{h(\sqrt{x+h-2} + \sqrt{x-2})}$$

* cancel out opposite terms

$$= \lim_{h\to 0}\frac{x+h-2-(x-2)}{h(\sqrt{x+h-2}+\sqrt{x-2})} = \lim_{h\to 0}\frac{\cancel{x}+h\cancel{-2}\cancel{-x}\cancel{+2}}{h(\sqrt{x+h-2}+\sqrt{x-2})}$$

** take out "h" GCF

$$= \lim_{h\to 0}\frac{\cancel{h}}{\cancel{h}(\sqrt{x+h-2}+\sqrt{x-2})}$$

* do lim $h\to 0$ and get "like terms"

$$= \lim_{h\to 0}\frac{1}{\sqrt{x+h-2}+\sqrt{x-2}} = \frac{1}{\sqrt{x-2}+\sqrt{x-2}}$$

$$f'(x) = \boxed{\frac{1}{2\sqrt{x-2}}}$$

You Try) find $f'(x)$ for $f(x) = \sqrt{x+4}$

Practice...

- Find the derivative of the following using the limit definition of derivative.

$$f(x) = \sqrt{x+4}$$

Practice) ANSWERS

Use the limit definition of derivative to Find $f'(x)$ for $f(x) = \sqrt{x+4}$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{\left(\sqrt{x+h+4} - \sqrt{x+4}\right)}{h} \frac{\left(\sqrt{x+h+4} + \sqrt{x+4}\right)^{*}}{\left(\sqrt{x+h+4} + \sqrt{x+4}\right)}$$

*multiply conjugate to top and bottom

$$\lim_{h \to 0} \frac{\left(\cancel{\sqrt{}}x+h+4\right)^{\cancel{2}*} + \cancel{\sqrt{x+h+4}\sqrt{x+4}}^{**} - \cancel{\sqrt{x+4}\sqrt{x+h+4}} - \left(\cancel{\sqrt{}}x+4\right)^{\cancel{2}*}}{h\left(\sqrt{x+h+4} + \sqrt{x+4}\right)}$$

** cancel out opposite terms

* square root and square are opposites

$$= \lim_{h \to 0} \frac{x+h+4-(x+4)}{h\left(\sqrt{x+h+4} + \sqrt{x+4}\right)} = \lim_{h \to 0} \frac{\cancel{x}^{**} + h + \cancel{4} - \cancel{x} - \cancel{4}}{h\left(\sqrt{x+h+4} + \sqrt{x+4}\right)}$$

* take out "h" GCF

$$= \lim_{h \to 0} \frac{\cancel{h}^{*}}{\cancel{h}\left(\sqrt{x+h+4} + \sqrt{x+4}\right)} = \lim_{h \to 0} \frac{1}{\underbrace{\sqrt{x+\cancel{h}+4} + \sqrt{x+4}}_{\sqrt{x+4}\sqrt{x+4}}}$$

≡* do $\lim_{h \to 0}$ to get "like terms"

$$= \boxed{\frac{1}{2\sqrt{x+4}}}$$

PREREQUISITE REFRESHER

You're off to lunch with a friend. You're going to split 1/3 of a pizza and 1/4 of a sub... evenly with your friend. How much pizza and sub do you get?

Your lunch =

$$\frac{\underset{\text{pizza}}{\frac{1}{3}} + \underset{\text{sub}}{\frac{1}{4}}}{2}$$

$$= \boxed{\underset{\text{pizza}}{\frac{1}{6}} + \underset{\text{sub}}{\frac{1}{8}}}$$

OR simpler way

$$\left(\frac{1}{3}\text{pz} + \frac{1}{4}\text{sub}\right)\cdot\frac{1}{2}$$

$$\cancel{2 \times \frac{1}{2}}$$

$$= \boxed{\frac{1}{6}\text{pz} + \frac{1}{8}\text{sub}}$$

Example 4:

Evaluate the derivative using the limit definition of derivatives. $\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$

- **Function:** $f(x) = \frac{1}{x+1}$

$$\lim_{h\to 0} \frac{\left(\frac{1}{x+h+1} - \frac{1}{x+1}\right)}{h} \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{1}{x+h+1} - \frac{1}{x+1}\right)$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{1}{(x+h+1)}\cdot\frac{(x+1)}{(x+1)}^{*} - \frac{1}{(x+1)}\cdot\frac{(x+h+1)}{(x+h+1)}^{*}\right)$$

* Get common denom -inator

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{x+1-1(x+h+1)}{(x+h+1)(x+1)}\right)$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{\cancel{x}+\cancel{1}-\cancel{x}-h-\cancel{1}^{*}}{(x+h+1)(x+1)}\right)$$

* cancel out terms

$$\lim_{h\to 0} \frac{-\cancel{h}^{**}}{\cancel{h}(x+h+1)(x+1)}$$

** take out "h" GCF

$$\lim_{h\to 0} \frac{-1}{(x+h+1)(x+1)}^{**} = \frac{-1}{(x+1)(x+1)} = \boxed{\frac{-1}{(x+1)^2}} = \boxed{\frac{-1}{x^2+2x+1}}$$

** Do $\lim\limits_{h\to 0}$ to get like terms

Practice...

- Find the derivative of the following using the limit definition of derivative.

$$f(x)=\frac{1}{x-3}$$

Practice) Using the limit definition of derivative, find $f'(x)$ for $f(x) = \frac{1}{x-3}$

ANSWERS

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\left(\frac{1}{x+h-3} - \frac{1}{x-3}\right) \cdot \frac{1}{h}}{(\cancel{h}) \cdot \frac{1}{\cancel{h}}}$$

$$= \lim_{h \to 0} \frac{1}{h}\left(\frac{1}{x+h-3} - \frac{1}{x-3}\right)$$

$$= \lim_{h \to 0} \frac{1}{h}\left(\frac{1}{(x+h-3)} \cdot \frac{(x-3)^*}{(x-3)} - \frac{1}{(x-3)} \cdot \frac{(x+h-3)^*}{(x+h-3)}\right)$$

* Get common denominator

$$= \lim_{h \to 0} \frac{1}{h}\left(\frac{(x-3) - (x+h-3)}{(x+h-3)(x-3)}\right)$$

$$= \lim_{h \to 0} \frac{1}{h}\left(\frac{\cancel{x} - \cancel{3} - \cancel{x} - h + \cancel{3}}{(x+h-3)(x-3)}\right)$$

** cancel out terms

$$= \lim_{h \to 0} \frac{1}{\cancel{h}}\left(\frac{-\cancel{h}}{(x+h-3)(x-3)}\right)$$

** take out the "h" GCF

$$= \lim_{h \to 0} \frac{-1}{(x+h-3)(x-3)} = \boxed{\frac{-1}{(x-3)^2} = f'(x)}$$

or $\boxed{\frac{-1}{x^2-6x+9} = f'(x)}$

** Do $\lim_{h \to 0}$ to get like terms