

KEY
F'18

Unit 2 Day 2 MATRICES

More MATRIX Applications & Start Leslie Matrices

waited until Day 3 to do to work more on ones like warm up

Warm Up Day 2

1. 1. A florist creates three special floral arrangements. One type uses three lilies. The second type uses two lilies and four carnations. The third uses two daisies and four carnations. Lilies cost \$2.35 each, carnations \$0.95 each, and daisies \$1.45 each.

- Write a matrix N to represent the number of each type of flower in each arrangement. *see below*
- Write a matrix C to represent the cost of each type of flower.
- Use matrix operations to find a matrix representing the cost of each type of arrangement.

2. Simplify.
$$\begin{bmatrix} -3 & 2a \\ a & 1 \\ 2 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & a \\ 2 & 3 \end{bmatrix}$$

and Look at re peared * info to see what to "scratch out" by putting in column of 1st + row of 2nd matrix because it's NOT part of the goal *

$(3 \times 3) \cdot (3 \times 1)$
Lilies CP cost *

Look at Goal: cost by type

a) Type 1
N = Type 2
Type 3

| | Lilies | Carn. | Daisies |
|--------|--------|-------|---------|
| Type 1 | 3 | 0 | 0 |
| Type 2 | 2 | 4 | 0 |
| Type 3 | 0 | 4 | 2 |

b) C =

| | Cost |
|---------|------|
| Lilies | 2.35 |
| Carn. | 0.95 |
| Daisies | 1.45 |

c) N · C

$$\begin{bmatrix} 3(2.35) + 0(0.95) + 0(1.45) \\ 2(2.35) + 4(0.95) + 0(1.45) \\ 0(2.35) + 4(0.95) + 2(1.45) \end{bmatrix}$$

| | |
|--------|------|
| Type 1 | 7.05 |
| Type 2 | 8.50 |

Warm Up Day 2

1. 1. A florist creates three special floral arrangements. One type uses three lilies. The second type uses two lilies and four carnations. The third uses two daisies and four carnations. Lilies cost \$2.35 each, carnations \$0.95 each, and daisies \$1.45 each.

- a. Write a matrix N to represent the number of each type of flower in each arrangement.
- b. Write a matrix C to represent the cost of each type of flower.
- c. Use matrix operations to find a matrix representing the cost of each type of arrangement.

2. Simplify.
$$\begin{bmatrix} -3 & 2a \\ a & 1 \\ 2 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & a \\ 2 & 3 \end{bmatrix}$$

2

Warm Up Answers

1. A florist creates three special floral arrangements. One type uses three lilies. The second type uses two lilies and four carnations. The third uses two daisies and four carnations. Lilies cost \$2.35 each, carnations \$0.95 each, and daisies \$1.45 each.

- a. Write a matrix N to represent the number of each type of flower in each arrangement.

$$N = \begin{matrix} & \begin{matrix} Lil & Cr & Da \end{matrix} \\ \begin{matrix} A1 \\ A2 \\ A3 \end{matrix} & \begin{bmatrix} 3 & 0 & 0 \\ 2 & 4 & 0 \\ 0 & 4 & 2 \end{bmatrix} \end{matrix}$$

- b. Write a matrix C to represent the cost of each type of flower.

$$C = \begin{matrix} & \begin{matrix} Lil \\ Cr \\ Da \end{matrix} \\ \begin{matrix} Cost (\$) \end{matrix} & \begin{bmatrix} 2.35 \\ 0.95 \\ 1.45 \end{bmatrix} \end{matrix}$$

- c. Use matrix operations to find a matrix representing the cost of each type of arrangement.

$$NC = \begin{matrix} & \begin{matrix} Cost (\$) \end{matrix} \\ \begin{matrix} Cost (\$) \end{matrix} & \begin{bmatrix} 3(2.35) + 0(0.95) + 0(1.45) \\ 2(2.35) + 4(0.95) + 0(1.45) \\ 0(2.35) + 4(0.95) + 2(1.45) \end{bmatrix} = \begin{matrix} \begin{matrix} A1 \\ A2 \\ A3 \end{matrix} & \begin{bmatrix} 7.05 \\ 8.50 \\ 6.70 \end{bmatrix} \end{matrix}_3$$

Warm Up Answers

2. Simplify.

$$\begin{bmatrix} -3 & 2a \\ a & 1 \\ 2 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & a \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} -3(1) + 2a(2) & -3a + 2a(3) \\ a(1) + 1(2) & a(a) + 1(3) \\ 2(1) + 0(2) & 2(a) + 0(3) \end{bmatrix}$$

$(3 \times 2) (2 \times 2)$

$$= \begin{bmatrix} -3 + 4a & 3a \\ a + 2 & a^2 + 3 \\ 2 & 2a \end{bmatrix}$$

3×2

4

**Questions About
last night's HW?**

**Packet p. 1
or Matrices Applications
Handout?**



Homework

- Finish Day 1 Slides Apps Handout
- Finish Day 2 Handout on Mixed Matrix Applications



Unit 2 Day 2 NOTES

***More MATRIX Applications
& Start Leslie Matrices***

Notes: Solving Systems of Equations with Matrices

Ex 1: Setting up and Solving Matrix Equations

Solve

$$5x - 4y = 8$$

$$1x + 2y = 6$$

A linear system can be written as a matrix equation $AX=B$

$$\begin{bmatrix} 5 & -4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$$

↑ ↑ ↑
Coefficient Variable Constant
matrix, matrix, matrix,
A X B

We'll solve for matrix X to find x and y.

Ex 1:

Solving Matrix Equations

Suppose $ax = b$. How do you solve for x ?

We cannot divide matrices, but we can multiply by the inverse.

A^{-1} is the inverse of matrix A .

We'll use it to find X .

$$AX = B$$

$$\cancel{A^{-1}A}X = A^{-1}B$$

$$X = \underbrace{A^{-1}B}$$

We can do this in the calc. ☺

Details on next slide →

WARNING! This is NOT a -1 exponent!!

Ex 1: Solving Matrix Equations in calculator

Solve

$$\begin{aligned} 5x - 4y &= 8 \\ 1x + 2y &= 6 \end{aligned}$$

$$\begin{bmatrix} 5 & -4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$$

↑
A

↑
B

In Calc, 2nd x^{-1} Edit & enter matrices A and B

Remember, $AX = B$

$$X = \underbrace{A^{-1}B}$$

On main screen,

2nd x^{-1} Enter (to get A)

x^{-1} (to get inverse $^{-1}$ on A)

2nd x^{-1} 2 Enter (to get B)

$$x = 2.857, \quad y = -1.571 \quad (2.857, -1.571)$$

Ex. 2 Solve using matrices

$$2x + 3y + z = -1$$

$$3x + 3y + z = 1$$

$$2x + 4y + z = -2$$

$$\begin{bmatrix} 2 & 3 & 1 \\ 3 & 3 & 1 \\ 2 & 4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}$$

↑
A

↑
B

$$AX = B$$

$$X = A^{-1}B$$

We can do this
in the calc. 😊

$$x = 2$$

$$y = -1$$

$$z = -2$$

$$(2, -1, -2)$$

You Try! Solve using matrices

Ex. 3 $7x + 3y = 11$

$$14x - 4y = 2$$

1st set up & line up variables

Ex. 4 $-x + 3y + 5z = -15$

$$\Rightarrow 2x + y + 0z = 1$$

$$-9x - 8y - 4z = 12$$

$$\begin{bmatrix} -1 & 3 & 5 \\ 2 & 1 & 0 \\ -9 & -8 & -4 \end{bmatrix}^{-1} \begin{bmatrix} -15 \\ 1 \\ 12 \end{bmatrix}$$

$$\begin{aligned} 3y + 5z &= x - 15 \\ 2x + y &= 1 \\ -9x - 8y &= 4z + 12 \end{aligned}$$

You Try! ANSWERS
Solve using matrices

Ex. 3 $7x + 3y = 11$
 $14x - 4y = 2$

$x = 5/7 = 0.71$

$y = 2$

$(5/7, 2)$
 $= (0.71, 2)$

Ex. 4 $-x + 3y + 5z = -15$
 $2x + y + 0z = 1$
 $-9x - 8y - 4z = 12$

$x = 4$

$y = -7$

$z = 2$

$(4, -7, 2)$

Hints for Applications:

*where you must
write your own
equations*

1. Read the problem and see if there are any "totals" given.
2. Write equations with these first.
3. See if there is anything else you can make an equation out of.
4. From your equations, set up your coefficient, variable, and constant matrix. Then solve!

$A^{-1} \cdot B$

Any time we need x, y, z, etc

Inverse Matrices and Systems

(on power point slide handout)

Ex 1) A linen shop has several tables of sheets and towels on special sale. The sheets are all priced the same, and so are the towels. Mario bought 3 sheets and 5 towels at a cost of \$137.50. Marco bought 4 sheets and 2 towels at a cost of \$118.00. Find the price of each item.

Relate:

| | | |
|-----------------------|------|--------|
| 3 sheets and 5 towels | cost | 137.50 |
| 4 sheets and 2 towels | cost | 118.00 |

$$\begin{aligned} 3x + 5y &= 137.50 \\ 4x + 2y &= 118.00 \end{aligned}$$

Define: Let x = the price of one sheet.
Let y = the price of one towel.

← GOAL

Write: $\begin{bmatrix} 3 & 5 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 137.50 \\ 118.00 \end{bmatrix}$

Let A = coefficient matrix and B = constant matrix. Do $A^{-1} * B$

$$= \begin{bmatrix} 22.50 \\ 14.00 \end{bmatrix}$$

Interpret:

The price of a sheet is \$22.50.
The price of a towel is \$14.00.

ANSWERS Inverse Matrices and Systems

(on power point slide handout)

Ex 1) A linen shop has several tables of sheets and towels on special sale. The sheets are all priced the same, and so are the towels. Mario bought 3 sheets and 5 towels at a cost of \$137.50. Marco bought 4 sheets and 2 towels at a cost of \$118.00. Find the price of each item.

Relate:

| | | |
|-----------------------|------|----------|
| 3 sheets and 5 towels | cost | \$137.50 |
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Define: Let x = the price of one sheet.
Let y = the price of one towel.

Write: $\begin{bmatrix} 3 & 5 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 137.50 \\ 118.00 \end{bmatrix}$

Let A = coefficient matrix and B = constant matrix. Do $A^{-1} * B$

The price of a sheet is \$22.50. The price of a towel is \$14.00.

Example 2

(on power point slide handout)

The sum of three numbers is 12. The first is five times the second and the sum of the first and third is 9. Find the numbers.

Let x = first number, y = second number, z = third number

$$x + y + z = 12$$

$$x = 5y$$

$$x + z = 9$$

$$x + y + z = 12$$

$$\Rightarrow x - 5y = 0$$

$$x + z = 9$$

The first is 15,
second is 3,
and third is -6

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -5 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 9 \end{bmatrix}$$

$$A \cdot X = B$$

$$X = A^{-1} \cdot B = \begin{bmatrix} 15 \\ 3 \\ -6 \end{bmatrix}$$

Example 2

Answers

The sum of three numbers is 12. The first is five times the second and the sum of the first and third is 9. Find the numbers.

Let x = first number, y = second number, z = third number

$$x + y + z = 12$$

$$x = 5y$$

$$x + z = 9$$

$$x + y + z = 12$$

$$x - 5y = 0$$

$$x + z = 9$$

$$(15, 3, -6)$$

You try:

1. My mom has three brothers. Together, their ages total 108. The youngest is 8 years less than the oldest. The middle one is four years older than the youngest. How old is each brother?
2. I have nickels, dimes, and quarters in my piggy bank. When I totaled it up last weekend, I had \$12.60. I remember I had 110 coins, and that there were only two more dimes than quarters. How many of each type did I have?



You try Answers:

(matrices on next slide)

1. My mom has three brothers. Together, their ages total 108. The youngest is 8 years less than the oldest. The middle one is four years older than the youngest. How old is each brother?

$$x + y + z = 108$$

$$x = z - 8$$

$$y = x + 4$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 108 \\ -8 \\ 4 \end{bmatrix} = \begin{bmatrix} 32 \\ 36 \\ 40 \end{bmatrix}$$

first is 40
middle is 36
youngest is 32

2. I have nickels, dimes, and quarters in my piggy bank. When I totaled it up last weekend, I had \$12.60. I remember I had 110 coins, and that there were only two more dimes than quarters. How many of each type did I have?

$$n + d + q = 110$$

$$.05n + .10d + .25q = 12.60$$

$$d = q + 2 \Rightarrow d - q = 2$$

$$\begin{bmatrix} 52 \\ 30 \\ 28 \end{bmatrix}$$



52 nickels
30 dimes
28 quarters

$$\begin{bmatrix} 1 & 1 & 1 \\ .05 & .10 & .25 \\ 0 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 110 \\ 12.60 \\ 2 \end{bmatrix}$$

test:
if oldest = 30,
then youngest = 22

$$x - z = -8$$

$$22 - 30 = -8 \checkmark$$

$$x + y + z = 108$$

$$x + 0y - z = -8$$

$$-x + y + 0z = 4$$

x = youngest bro's age
y = middle bro's age
z = oldest bro's age

You Try Matrix work:

1. (32, 36, 40) coefficient matrix: $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$
constant matrix: $\begin{bmatrix} 108 \\ -8 \\ 4 \end{bmatrix}$
2. $\begin{bmatrix} 52 \\ 30 \\ 28 \end{bmatrix}$ Nickels
Dimes
Quarters coefficient matrix: $\begin{bmatrix} 1 & 1 & 1 \\ .05 & .10 & .25 \\ 0 & 1 & -1 \end{bmatrix}$
constant matrix: $\begin{bmatrix} 110 \\ 12.60 \\ 2 \end{bmatrix}$

You try!

3. Janet is spending the allowance she has saved on clothes. If she buys 3 shirts, 2 skirts, and 4 pairs of jeans, she will spend \$292. If she buys 4 shirts, 1 skirt, and 3 pairs of jeans, she will spend \$252. If jeans cost \$4 more than skirts, find the price of each item.
4. At Morgan's Fine Cuisine, meals are served a la carte. That is, each item on the menu is priced separately. Jackie and Ted Paris went to celebrate their anniversary. Jackie ordered prime rib, 2 side dishes, and a roll. Ted ordered prime rib, 3 side dishes, and 2 rolls. Jackie's meal cost \$36 while Ted's cost \$44. If the prime rib is three times as expensive as a side dish, what is the cost of each item?