ICM ~ Unit 3 ~ Day 2

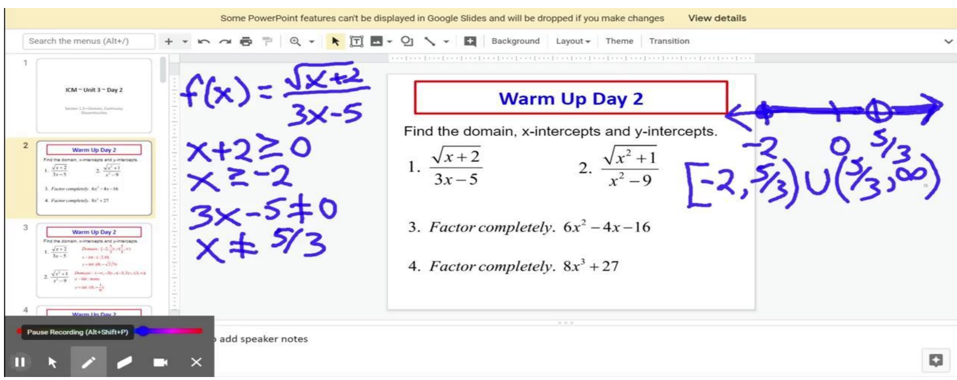
Section 1.2—Domain, Continuity, Discontinuities

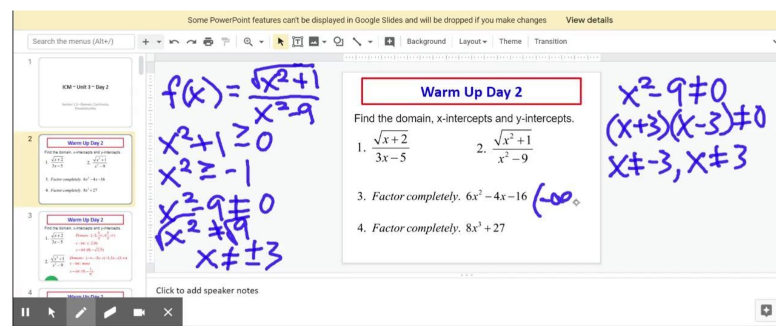
Warm Up Day 2

Find the domain, x-intercepts and y-intercepts.

1. $\dfrac{\sqrt{x+2}}{3x-5}$

2. $\dfrac{\sqrt{x^2+1}}{x^2-9}$





3. *Factor completely.* $6x^2-4x-16$

4. *Factor completely.* $8x^3+27$

Warm Up Day 2

Find the domain, x-intercepts and y-intercepts.

1. $\dfrac{\sqrt{x+2}}{3x-5}$

$Domain: [-2,\frac{5}{3})\cup(\frac{5}{3},\infty)$

$x-\text{int}: (-2,0)$

$y-\text{int}: (0,-\sqrt{2}/5)$

2. $\dfrac{\sqrt{x^2+1}}{x^2-9}$

$Domain: (-\infty,-3)\cup(-3,3)\cup(3,\infty)$

$x-\text{int}: none$

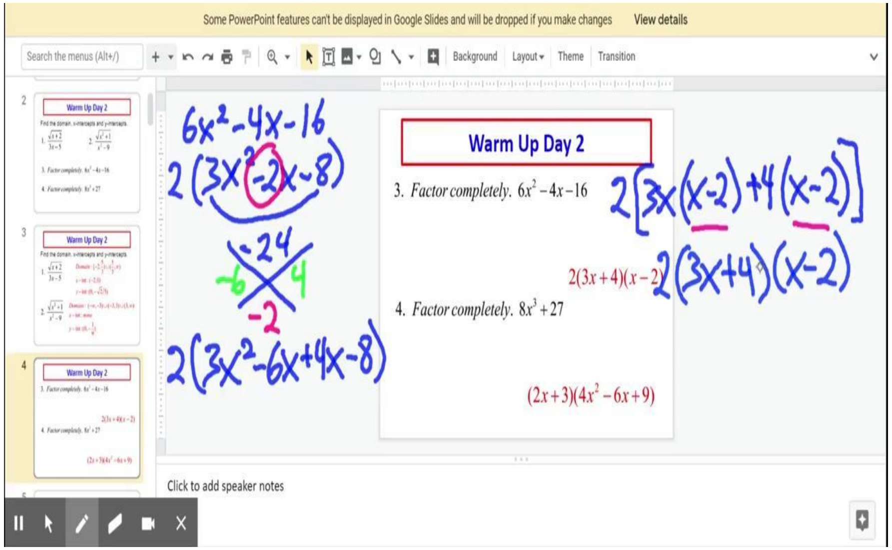
$y-\text{int}: (0,-\frac{1}{9})$

Warm Up Day 2

3. *Factor completely.* $6x^2 - 4x - 16$

$$2(3x+4)(x-2)$$

4. *Factor completely.* $8x^3 + 27$



$$(2x+3)(4x^2-6x+9)$$

Homework Questions?

Tonight's Homework

Packet p. 2-3

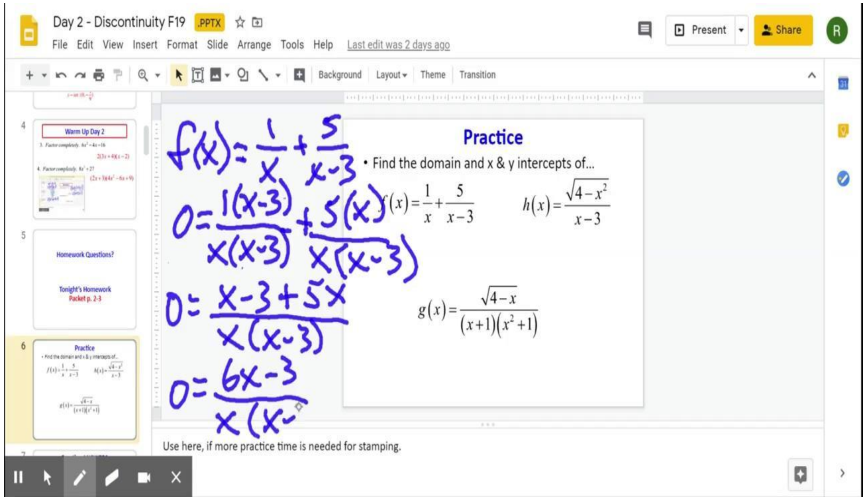
Practice

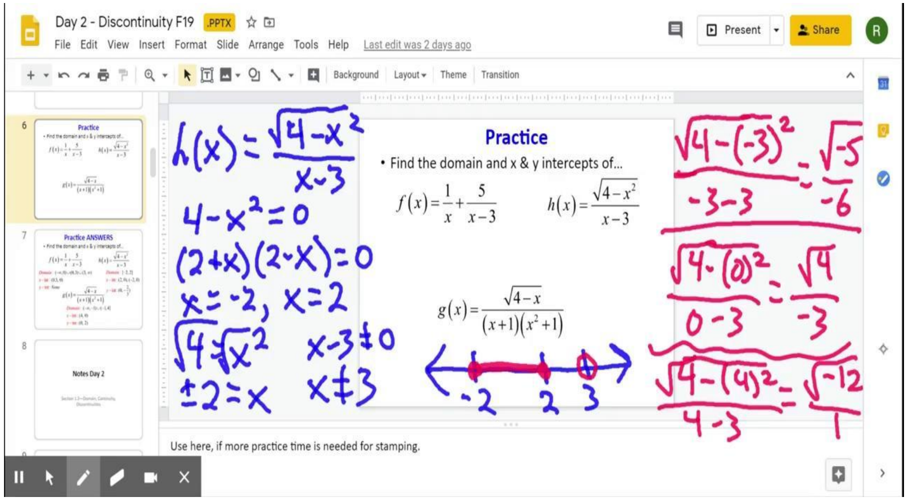
- Find the domain and x & y intercepts of…

$$f(x)=\frac{1}{x}+\frac{5}{x-3}$$

$$h(x)=\frac{\sqrt{4-x^2}}{x-3}$$

$$g(x)=\frac{\sqrt{4-x}}{(x+1)(x^2+1)}$$





Practice ANSWERS

- Find the domain and x & y intercepts of…

$$f(x) = \frac{1}{x} + \frac{5}{x-3}$$

$$Domain: \ (-\infty, 0) \cup (0, 3) \cup (3, \infty)$$

$$x - \text{int}: (0.5,\ 0)$$

$$y - \text{int}: None$$

$$h(x) = \frac{\sqrt{4-x^2}}{x-3}$$

$$Domain: [-2,\ 2]$$

$$x - \text{int}: (2,\ 0), (-2,\ 0)$$

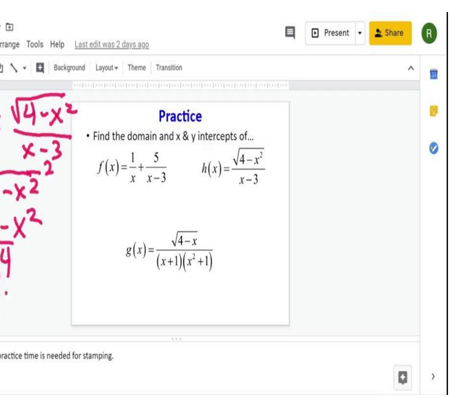
$$y - \text{int}: (0,\ -\frac{2}{3})$$

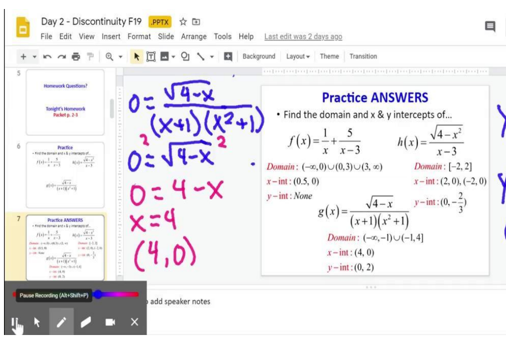
$$g(x) = \frac{\sqrt{4-x}}{(x+1)(x^2+1)}$$

$$Domain: \ (-\infty, -1) \cup (-1, 4]$$

$$x - \text{int}: (4,\ 0)$$

$$y - \text{int}: (0,\ 2)$$





Notes Day 2

Section 1.2—Domain, Continuity, Discontinuities

Defining Continuity

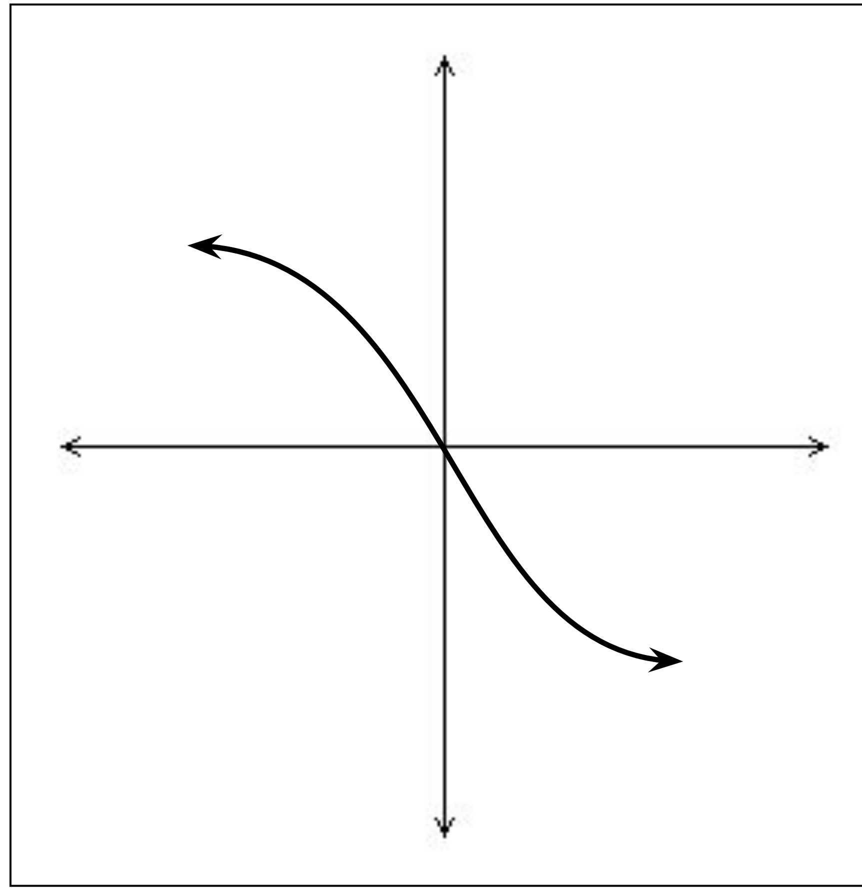
A function is **<u>continuous</u>** at a point if the graph does not come apart at that point.

Try graphing these:

- Ex: $y = -x^3$
- Ex: $f(x) = e^{2x} + 7$
- Ex: $g(x) = |x – 6|$
- Ex: $h(x) = \sqrt{x+2}$

A function is **continuous** at all “x-values” if . . .

- There are NO breaks in the curve.



Notice: You can trace the entire graph without lifting your pencil!

Different Types of **discontinuities:**

1. Removable – also referred to as a "*hole*"

2. Nonremovable

   a. "Jump" (occur in piecewise functions or greatest integer)

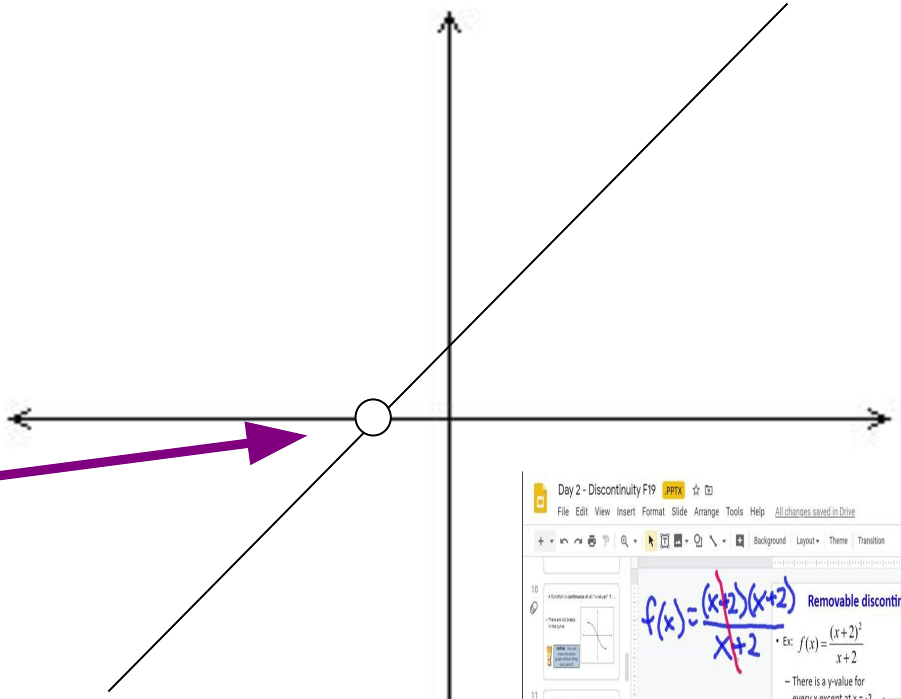
   b. Vertical Asymptote (infinite discontinuity)

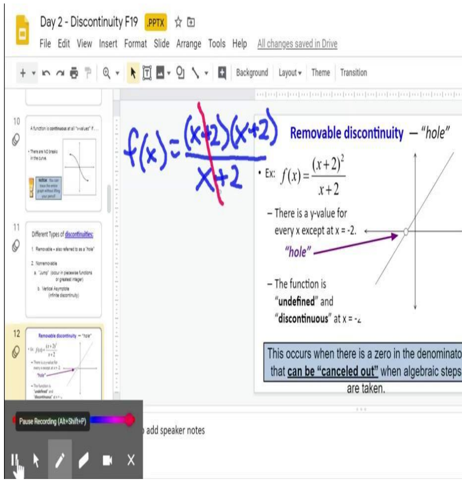
Removable discontinuity — *"hole"*

- Ex: $f(x) = \dfrac{(x+2)^2}{x+2}$
  - There is a y-value for every x except at x = -2.

    "hole"

  - The function is "**undefined**" and "**discontinuous**" at x = -2

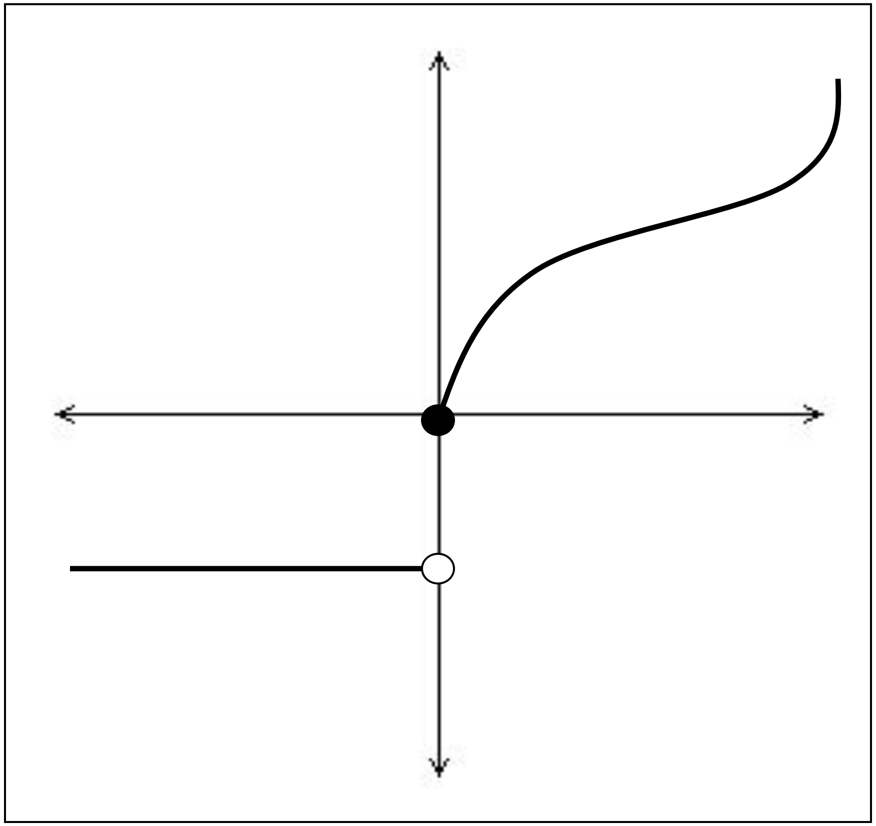




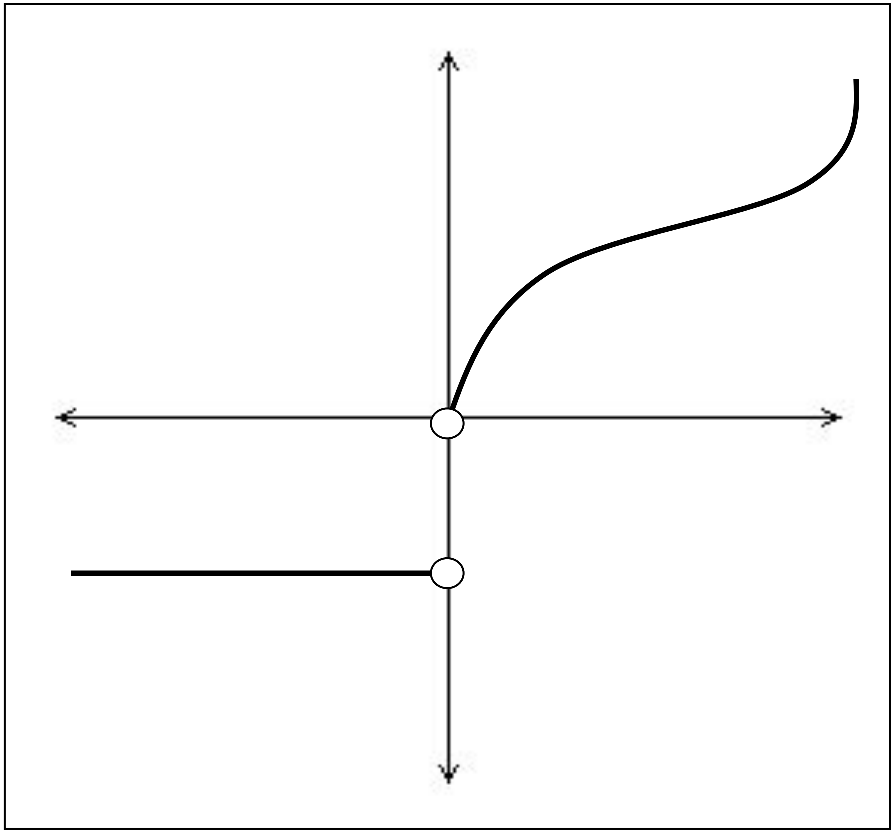
This occurs when there is a zero in the denominator that **<u>can be "canceled out"</u>** when algebraic steps are taken.

Jump Discontinuity – the curve "jumps" from one y-value to the next (NONREMOVABLE)

Notice: this graph still has a y-value for every x.



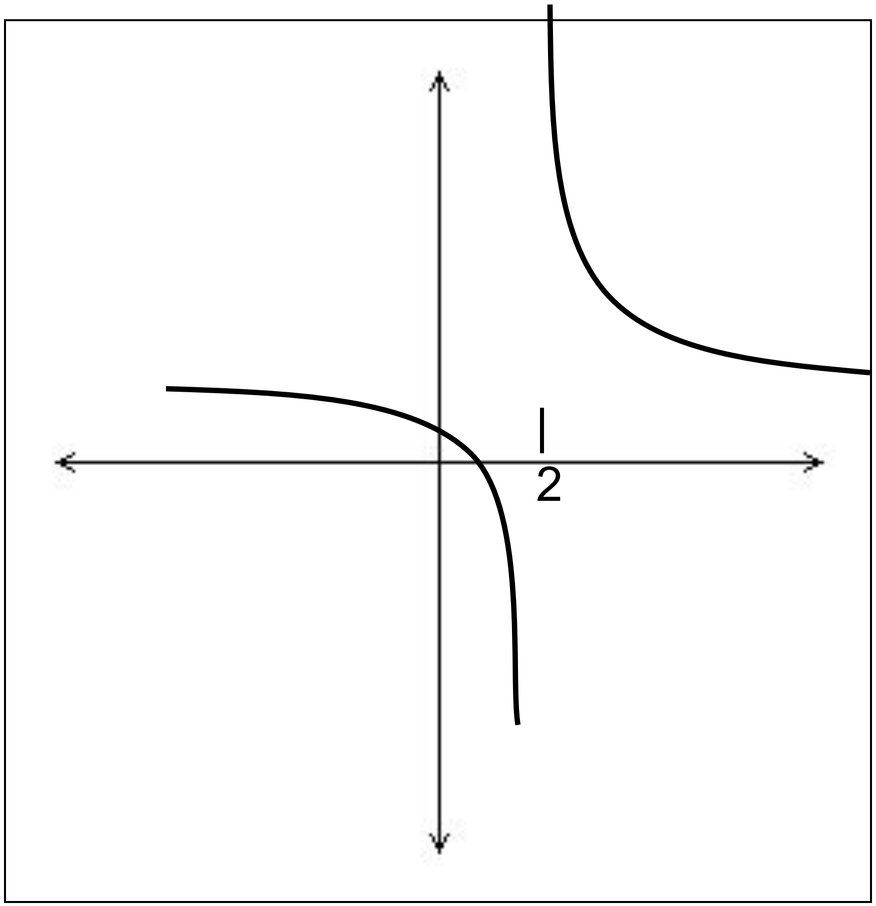
Notice: this one doesn't



Nonremovable Discontinunity

(Vertical Asymptote)—Infinite Discontinuity

- Ex: $f(x) = \dfrac{x+3}{x-2}$



There is a zero in the denominator of a function that **<u>cannot be "canceled out"</u>** through algebra.

Finding Discontinuities Summary

- To find vertical asymptotes and holes, factor the problem and simplify.
  - **If the factor in the bottom cancels out, it gives us a hole.**
    - Removable $f(x) = \frac{x+2}{(x+2)(x-1)}$

So the hole occurs at x = -2

$(-2, -\frac{1}{3})$

How do you find the y-value of the hole?

Simplify and substitute the x-value into the **remaining** equation!

$$f(-2) = \frac{1}{-2-1}$$

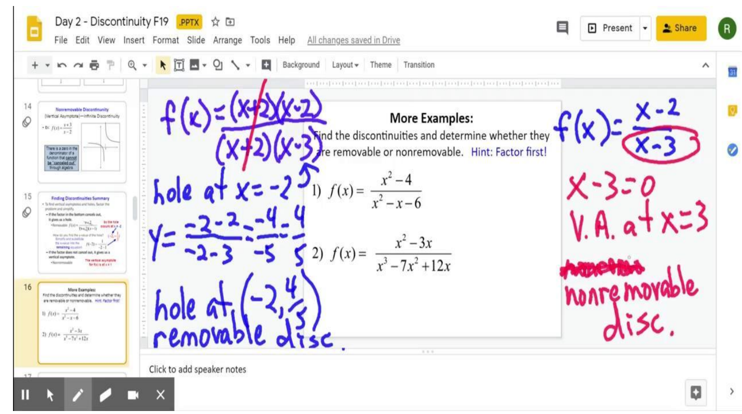
  - **If the factor does not cancel out, it gives us a vertical asymptote.**
    - Nonremovable

The vertical asymptote for f(x) is at x = 1

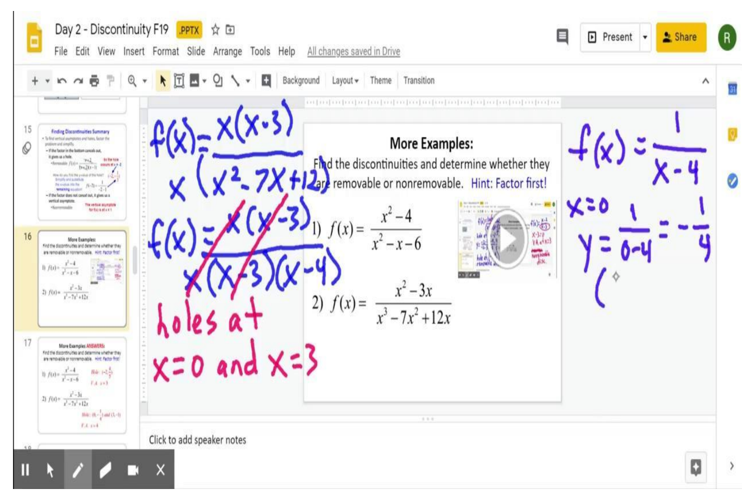
More Examples:

Find the discontinuities and determine whether they are removable or nonremovable. Hint: Factor first!

1) $f(x)=\dfrac{x^2-4}{x^2-x-6}$



2) $f(x)=\dfrac{x^2-3x}{x^3-7x^2+12x}$



More Examples ANSWERS:

Find the discontinuities and determine whether they are removable or nonremovable. Hint: Factor first!

1) $f(x) = \dfrac{x^2 - 4}{x^2 - x - 6}$

$Hole:\ (-2, \frac{4}{5})$

$V.A.\quad x = 3$

2) $f(x) = \dfrac{x^2 - 3x}{x^3 - 7x^2 + 12x}$

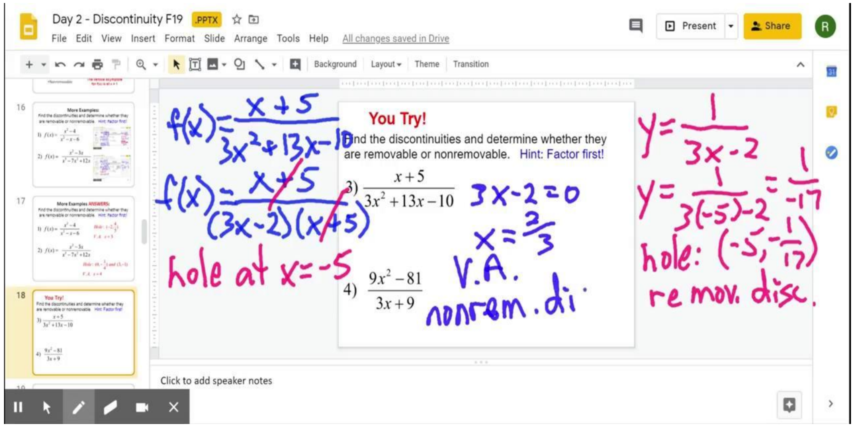
$Hole:\ (0, -\frac{1}{4})\ and\ (3, -1)$

$V.A.\quad x = 4$

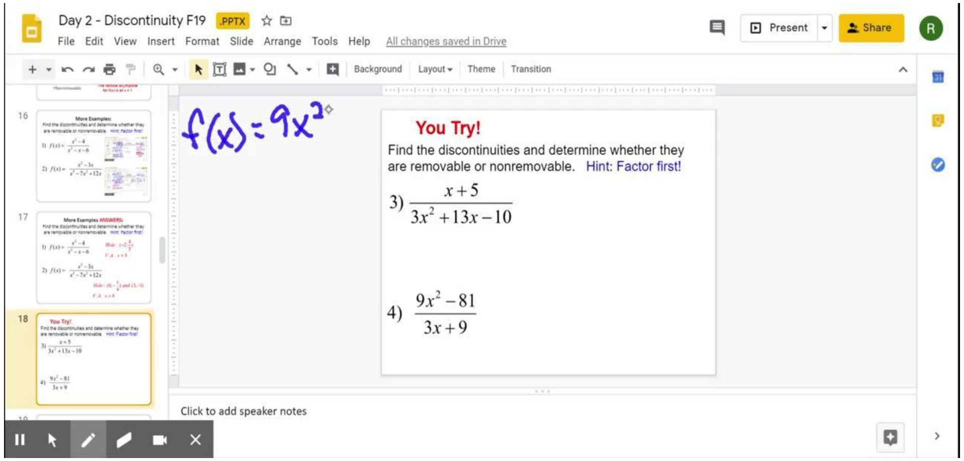
You Try!

Find the discontinuities and determine whether they are removable or nonremovable. Hint: Factor first!

3) $\dfrac{x+5}{3x^2+13x-10}$



4) $\dfrac{9x^2-81}{3x+9}$



You Try! ANSWERS

Find the discontinuities and determine whether they are removable or nonremovable. Hint: Factor first!

3) $$\frac{x+5}{3x^2+13x-10} = \frac{x+5}{(x+5)(3x-2)}$$

$$= \frac{1}{(3x-2)}$$

Hole: $(-5, -\frac{1}{17})$

V.A. $x = \frac{2}{3}$

4) $$\frac{9x^2-81}{3x+9} = \frac{9(x+3)(x-3)}{3(x+3)}$$

$$= 3(x-3)$$

Hole: $(-3, -18)$

V.A. *None*

Student Practice:

A) Classify the function as continuous or discontinuous.

B) If discontinuous, specify the type of discontinuity and where it exists.

C) State the domain and x & y intercepts.

1) $f(x) = (x+3)(x-2)$

2) $f(x) = \dfrac{x^2 - 4}{x - 2}$

Student Practice: ANSWERS

A) Classify the function as continuous or discontinuous.

B) If discontinuous, specify the type of discontinuity and where it exists.

C) State the domain and x & y intercepts.

1) $f(x) = (x+3)(x-2)$

Continuous

Domain: $(-\infty, \infty)$

$x-\text{int}: (-3, 0), (2, 0)$

$y-\text{int}: (0, -6)$

2) $f(x) = \dfrac{x^2 - 4}{x - 2}$

Discontinuous – removable (hole)

Hole at (2, 4)

Domain: $(-\infty, 2) \cup (2, \infty)$

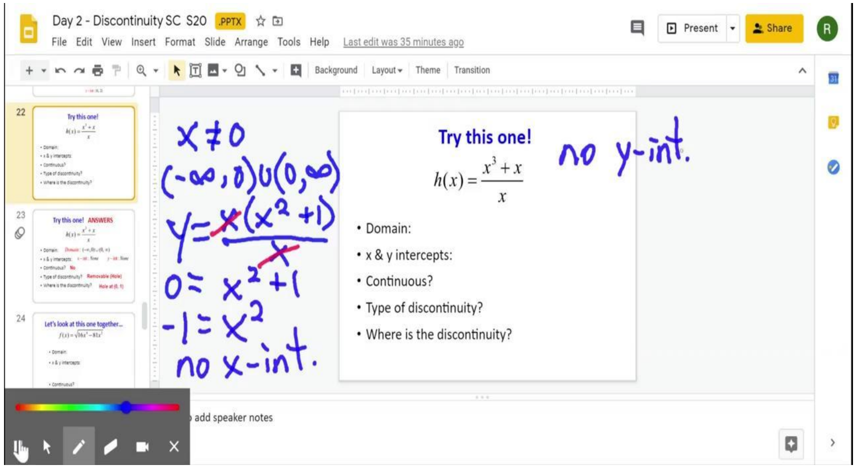
$x-\text{int}: (-2, 0)$

$y-\text{int}: (0, 2)$

Try this one!

$$h(x) = \frac{x^3 + x}{x}$$

- Domain:
- x & y intercepts:
- Continuous?
- Type of discontinuity?
- Where is the discontinuity?



Try this one! ANSWERS

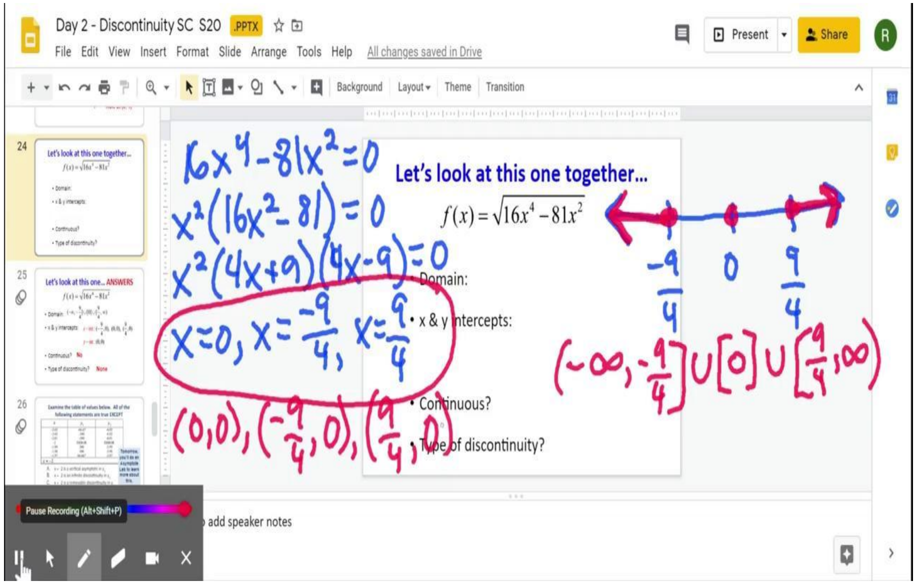
$$h(x) = \frac{x^3 + x}{x}$$

- Domain: $Domain: \ (-\infty, 0) \cup (0, \ \infty)$
- x & y intercepts: $x - \text{int}: None$ $y - \text{int}: None$
- Continuous? **No**
- Type of discontinuity? **Removable (Hole)**
- Where is the discontinuity? **Hole at (0, 1)**

Let's look at this one together…

$$f(x)=\sqrt{16x^4-81x^2}$$

- Domain:
- x & y intercepts:



- Continuous?
- Type of discontinuity?

Let's look at this one… ANSWERS

$$f(x)=\sqrt{16x^4-81x^2}$$

- Domain: $(-\infty,-\frac{9}{4}]\cup[0]\cup[\frac{9}{4},\infty)$
- x & y intercepts: $x-\text{int}:(-\frac{9}{4},0),\ (0,0),\ (\frac{9}{4},0)$

  $y-\text{int}:(0,0)$
- Continuous? **No**
- Type of discontinuity? **None**

Examine the table of values below. All of the following statements are true EXCEPT

x	y_1	y_2
-2.03	-66.67	-4.03
-2.02	-100	-4.02
-2.01	-200	-4.01
-2	ERROR	ERROR
-1.99	200	-3.99
-1.98	100	-3.98
-1.97	66.667	-3.97
$x = -2$		

A. x = -2 is a vertical asymptote in y_1

B. x = -2 is an infinite discontinuity in y_1

C. x = -2 is a removable discontinuity in y_2

D. x = -2 is a vertical asymptote in y_2 ⇐

Tomorrow, you'll do an Asymptote Lab to learn more about this.

Looking at y_2, the y's are in order, but y = -4 was skipped, so there is a Removable Discontinuity (Hole) at (-2, -4)

Definition of Degree

- **Degree of a polynomial in one variable:** the value of the greatest exponent

Ex: $f(x) = 4x^2 + 9x + 8$

Degree: 2

Ex: $g(x) = -5x^3 + 6x^2 + 4x$

Degree: 3

- **Degree can help us determine the horizontal asymptote of rational functions...**

Horizontal Asymptotes

For horizontal asymptotes, think BOSTON for ***polynomials***! Looking at the degree of top & bottom…

Bottom > Top

y=**O**

Same = ratio

Top > Bottom

↑ **O** No HA.

N

$$f(x) = \frac{2x}{x^2 + 3x} \qquad H.A.: y = 0$$

$$g(x) = \frac{2x^3}{5x^3 + 4x^2} \qquad H.A.: y = \frac{2}{5}$$

$$h(x) = \frac{5x^2}{7x + 3} \qquad No\ H.A.$$

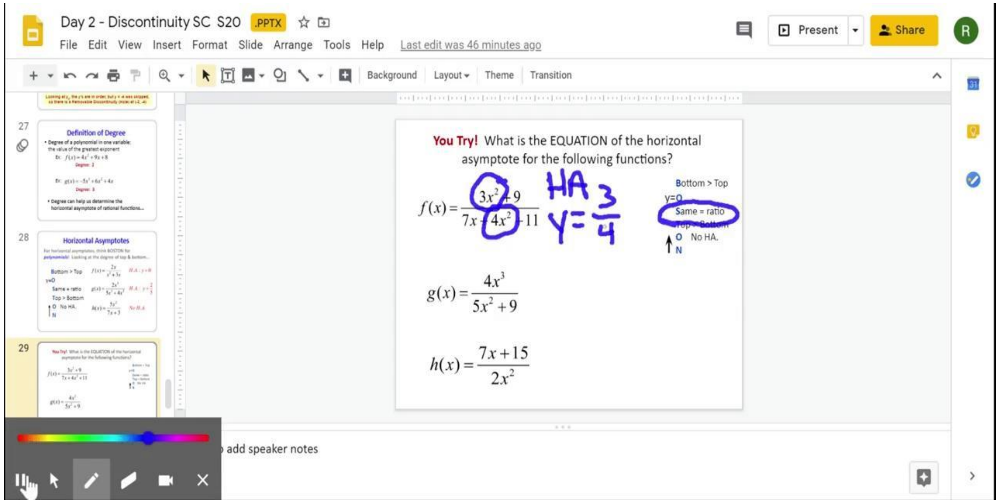
You Try! What is the EQUATION of the horizontal asymptote for the following functions?

$$f(x) = \frac{3x^2 + 9}{7x + 4x^2 + 11}$$

Bottom > Top
y=**O**
Same = ratio
Top > Bottom
↑ **O** No HA.
N

$$g(x) = \frac{4x^3}{5x^2 + 9}$$

$$h(x) = \frac{7x + 15}{2x^2}$$



You Try! What is the EQUATION of the horizontal asymptote for the following functions?

Bottom > Top

y=**O**

Same = ratio

Top > Bottom

O No HA.

↑ **N**

$$f(x)=\frac{3x^2+9}{7x+4x^2+11}$$

$H.A.: y = 3/4$

$$g(x)=\frac{4x^3}{5x^2+9}$$

$H.A.: none$

$$h(x)=\frac{7x+15}{2x^2}$$

$H.A.: y=0$

You Try: True or False

1) The graph of function f is defined as the set of all points $(x, f(x))$ where x is in the domain of f. Justify your answer.

2) If a function is not continuous, then the domain cannot be all real numbers.

True or False ANSWERS

1) The graph of function f is defined as the set of all points $(x, f(x))$ where x is in the domain of f. Justify your answer.

True! This is the definition of a function.

2) If a function is not continuous, then the domain cannot be all real numbers.

False! It could be a piecewise function.

Is this continuous?

State whether the scenario is continuous or discontinuous.

- A) Outdoor temperature as a function of time.

  Continuous

- B) Number of soft drinks sold at a ballpark as a function of outdoor temp. **Discontinuous**

- C) Your hair length as a function of days in a year

  Continuous

Homework Day 2

- Packet p. 2-3