

# ICM ~ Unit 3 ~ Day 2

Section 1.2—Domain, Continuity,  
Discontinuities

# Warm Up Day 2

Find the domain, x-intercepts and y-intercepts.

1.  $\frac{\sqrt{x+2}}{3x-5}$

2.  $\frac{\sqrt{x^2+1}}{x^2-9}$

**Warm Up Day 2**

Find the domain, x-intercepts and y-intercepts.

- $\frac{\sqrt{x+2}}{3x-5}$
- $\frac{\sqrt{x^2+1}}{x^2-9}$
- Factor completely.  $6x^2 - 4x - 16$
- Factor completely.  $8x^3 + 27$

Handwritten notes for problem 1:  
 $f(x) = \frac{\sqrt{x+2}}{3x-5}$   
 $x+2 \geq 0 \Rightarrow x \geq -2$   
 $3x-5 \neq 0 \Rightarrow x \neq 5/3$   
 Domain:  $[-2, 5/3) \cup (5/3, \infty)$

**Warm Up Day 2**

Find the domain, x-intercepts and y-intercepts.

- $\frac{\sqrt{x+2}}{3x-5}$
- $\frac{\sqrt{x^2+1}}{x^2-9}$
- Factor completely.  $6x^2 - 4x - 16$
- Factor completely.  $8x^3 + 27$

Handwritten notes for problem 2:  
 $f(x) = \frac{\sqrt{x^2+1}}{x^2-9}$   
 $x^2+1 \geq 0 \Rightarrow x^2 \geq -1$   
 $x^2 \neq 9 \Rightarrow x \neq \pm 3$   
 Domain:  $x \neq -3, x \neq 3$   
 $x^2 - 9 \neq 0 \Rightarrow (x+3)(x-3) \neq 0$

3. Factor completely.  $6x^2 - 4x - 16$

4. Factor completely.  $8x^3 + 27$

# Warm Up Day 2

Find the domain, x-intercepts and y-intercepts.

1. 
$$\frac{\sqrt{x+2}}{3x-5}$$

*Domain* :  $[-2, \frac{5}{3}) \cup (\frac{5}{3}, \infty)$

*x-int* :  $(-2, 0)$

*y-int* :  $(0, -\sqrt{2}/5)$

2. 
$$\frac{\sqrt{x^2+1}}{x^2-9}$$

*Domain* :  $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

*x-int* : *none*

*y-int* :  $(0, -\frac{1}{9})$

# Warm Up Day 2

3. Factor completely.  $6x^2 - 4x - 16$

$$2(3x + 4)(x - 2)$$

4. Factor completely.  $8x^3 + 27$

$$(2x + 3)(4x^2 - 6x + 9)$$

The screenshot shows a Google Slides presentation with three slides. The first slide (slide 2) shows the problem for question 3:  $6x^2 - 4x - 16$ . Handwritten work shows the expression being factored as  $2(3x^2 - 2x - 8)$ , with a red circle around  $-2x$ . A cross is drawn over  $-6$  and  $4$ , and  $-2$  is written below. The final factored form is  $2(3x^2 - 6x + 4x - 8)$ . The second slide (slide 3) shows the problem for question 4:  $8x^3 + 27$ . Handwritten work shows the expression being factored as  $2[3x(x-2) + 4(x-2)]$ , which simplifies to  $2(3x+4)(x-2)$ . The final factored form is  $(2x+3)(4x^2 - 6x + 9)$ . The third slide (slide 4) shows the problem for question 3:  $6x^2 - 4x - 16$ . Handwritten work shows the expression being factored as  $2(3x^2 - 2x - 8)$ , with a red circle around  $-2x$ . A cross is drawn over  $-6$  and  $4$ , and  $-2$  is written below. The final factored form is  $2(3x^2 - 6x + 4x - 8)$ .

**Homework Questions?**

**Tonight's Homework**

**Packet p. 2-3**

# Practice

- Find the domain and x & y intercepts of...

$$f(x) = \frac{1}{x} + \frac{5}{x-3} \qquad h(x) = \frac{\sqrt{4-x^2}}{x-3}$$

$$g(x) = \frac{\sqrt{4-x}}{(x+1)(x^2+1)}$$

Day 2 - Discontinuity F19 PPTX ☆

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Background Layout Theme Transition

4 Warm Up Day 2

5 Homework Questions?

6 Practice

Find the domain and x & y intercepts of...

$f(x) = \frac{1}{x} + \frac{5}{x-3}$   
 $0 = \frac{1(x-3) + 5(x)}{x(x-3)}$   
 $0 = \frac{x-3+5x}{x(x-3)}$   
 $0 = \frac{6x-3}{x(x-3)}$

$h(x) = \frac{\sqrt{4-x^2}}{x-3}$   
 $g(x) = \frac{\sqrt{4-x}}{(x+1)(x^2+1)}$

Use here, if more practice time is needed for stamping.

Day 2 - Discontinuity F19 PPTX ☆

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Background Layout Theme Transition

6 Practice

7 Practice ANSWERS

8 Notes Day 2

Find the domain and x & y intercepts of...

$h(x) = \frac{\sqrt{4-x^2}}{x-3}$   
 $4-x^2=0$   
 $(2+x)(2-x)=0$   
 $x=-2, x=2$   
 $\sqrt{4-x^2} \ge 0$   
 $\pm 2 = x \quad x \neq 3$

$f(x) = \frac{1}{x} + \frac{5}{x-3}$   
 $h(x) = \frac{\sqrt{4-x^2}}{x-3}$   
 $g(x) = \frac{\sqrt{4-x}}{(x+1)(x^2+1)}$

$\frac{\sqrt{4-(-3)^2}}{-3-3} = \frac{\sqrt{-5}}{-6}$   
 $\frac{\sqrt{4-(0)^2}}{0-3} = \frac{\sqrt{4}}{-3}$   
 $\frac{\sqrt{4-(4)^2}}{4-3} = \frac{\sqrt{-12}}{1}$

Use here, if more practice time is needed for stamping.

# Practice ANSWERS

- Find the domain and x & y intercepts of...

$$f(x) = \frac{1}{x} + \frac{5}{x-3}$$

$$h(x) = \frac{\sqrt{4-x^2}}{x-3}$$

*Domain:*  $(-\infty, 0) \cup (0, 3) \cup (3, \infty)$

*Domain:*  $[-2, 2]$

*x-int:*  $(0.5, 0)$

*x-int:*  $(2, 0), (-2, 0)$

*y-int:* None

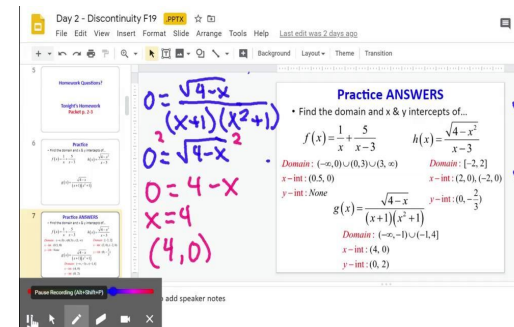
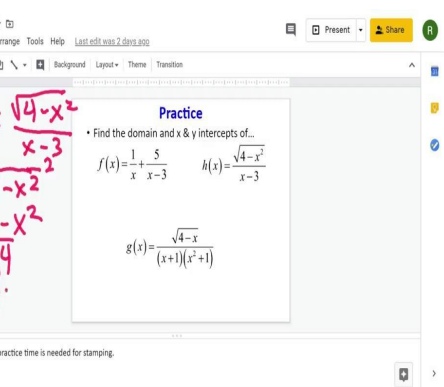
*y-int:*  $(0, -\frac{2}{3})$

$$g(x) = \frac{\sqrt{4-x}}{(x+1)(x^2+1)}$$

*Domain:*  $(-\infty, -1) \cup (-1, 4]$

*x-int:*  $(4, 0)$

*y-int:*  $(0, 2)$



# Notes Day 2

Section 1.2—Domain, Continuity,  
Discontinuities



# Defining Continuity

A function is **continuous** at a point if the graph does not come apart at that point.

*Try graphing these:*

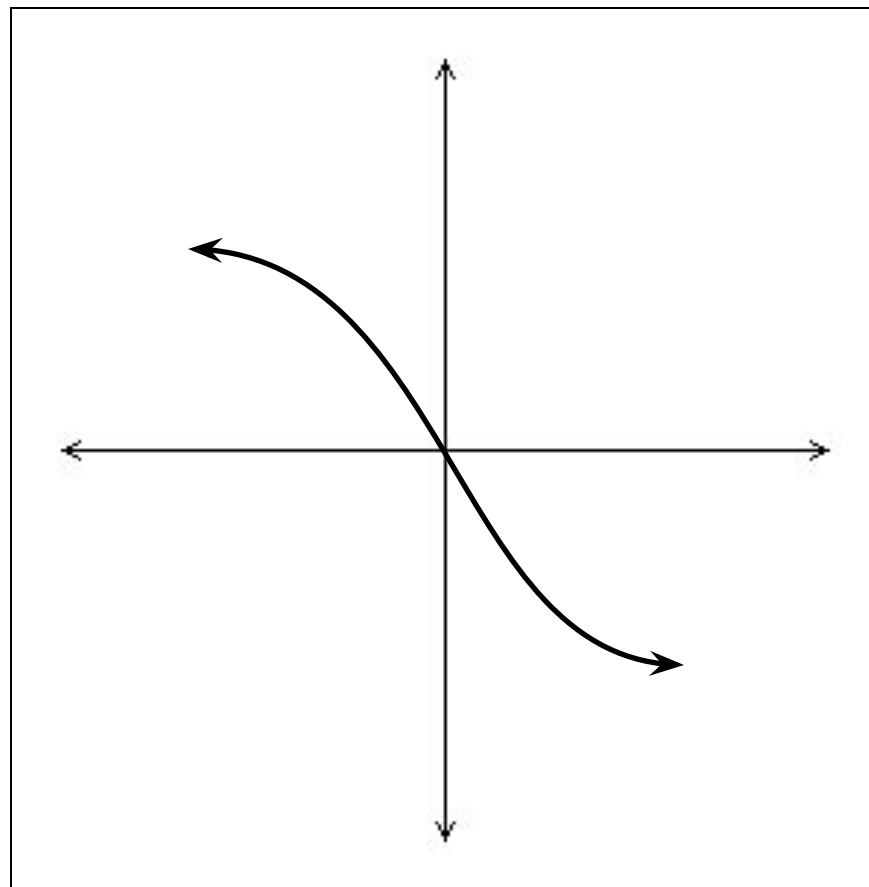
- Ex:  $y = -x^3$
- Ex:  $f(x) = e^{2x} + 7$
- Ex:  $g(x) = |x - 6|$
- Ex:  $h(x) = \sqrt{x + 2}$

A function is **continuous** at all “x-values” if . . .

- There are NO breaks in the curve.



**Notice:** You can trace the entire graph without lifting your pencil!



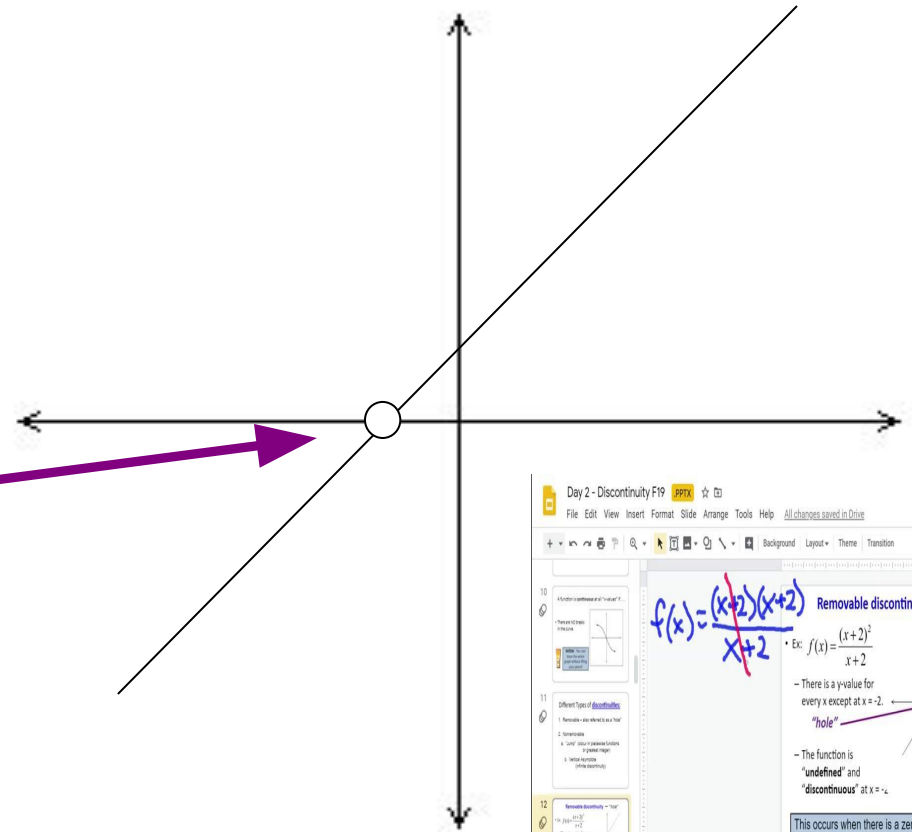
# Different Types of discontinuities:

1. Removable – also referred to as a “*hole*”
  
2. Nonremovable
  - a. “Jump” (occur in piecewise functions or greatest integer)
  
  - b. Vertical Asymptote  
(infinite discontinuity)

# Removable discontinuity — “hole”

- Ex:  $f(x) = \frac{(x+2)^2}{x+2}$ 
  - There is a y-value for every x except at  $x = -2$ .

“hole”



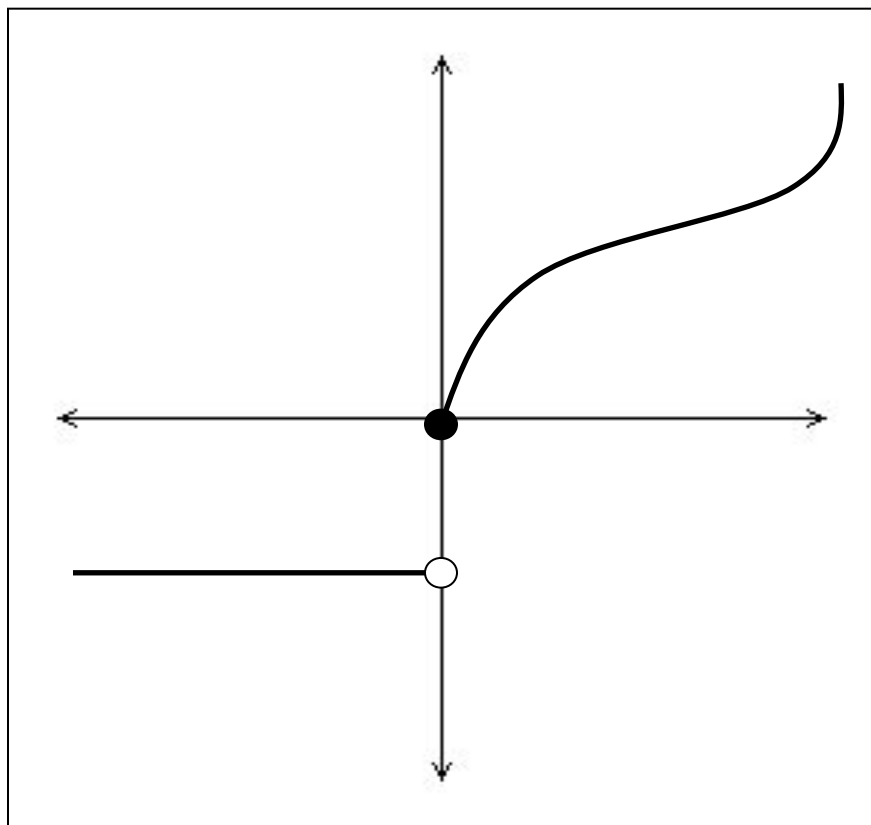
- The function is “undefined” and “discontinuous” at  $x = -2$

A screenshot of a presentation slide titled "Removable discontinuity — 'hole'". The slide shows the function  $f(x) = \frac{(x+2)(x+2)}{x+2}$  with the  $(x+2)$  in the denominator crossed out. It includes the example  $f(x) = \frac{(x+2)^2}{x+2}$  and a graph of the function with a hole at  $x = -2$ . The slide text explains that there is a y-value for every x except at  $x = -2$ , and that the function is "undefined" and "discontinuous" at  $x = -2$ . A note at the bottom states: "This occurs when there is a zero in the denominator that can be 'canceled out' when algebraic steps are taken." The screenshot also shows a presentation interface with a toolbar and a "Pause Recording" button.

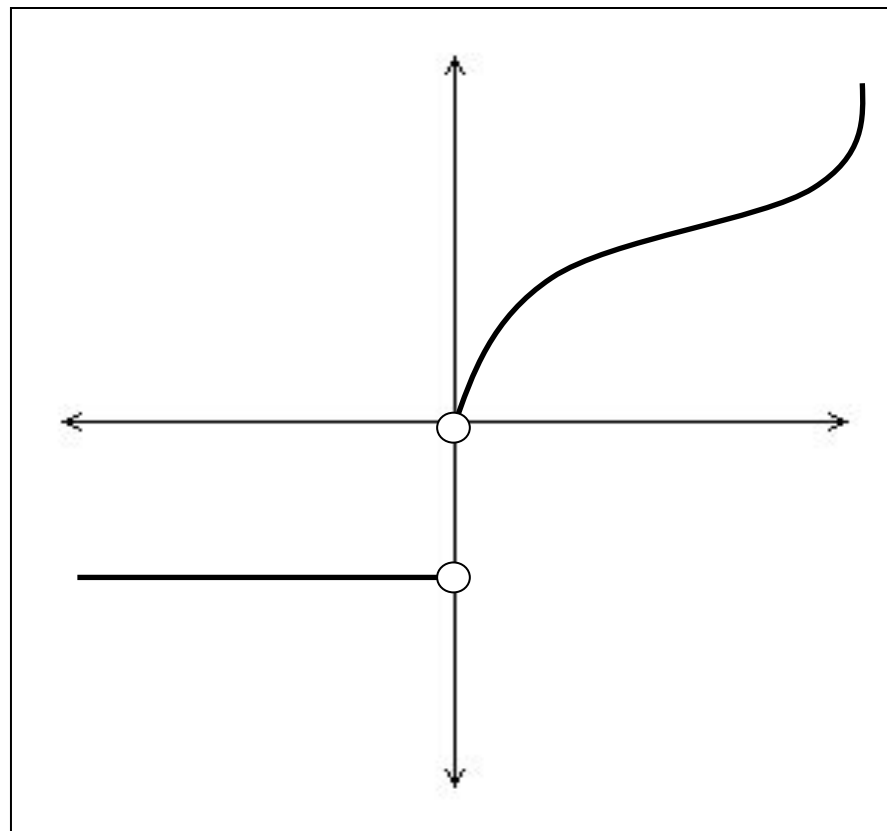
This occurs when there is a zero in the denominator that can be “canceled out” when algebraic steps are taken.

**Jump Discontinuity** – the curve “jumps” from one y-value to the next (NONREMOVABLE)

**Notice**: this graph still has a y-value for every x.



**Notice**: this one doesn't

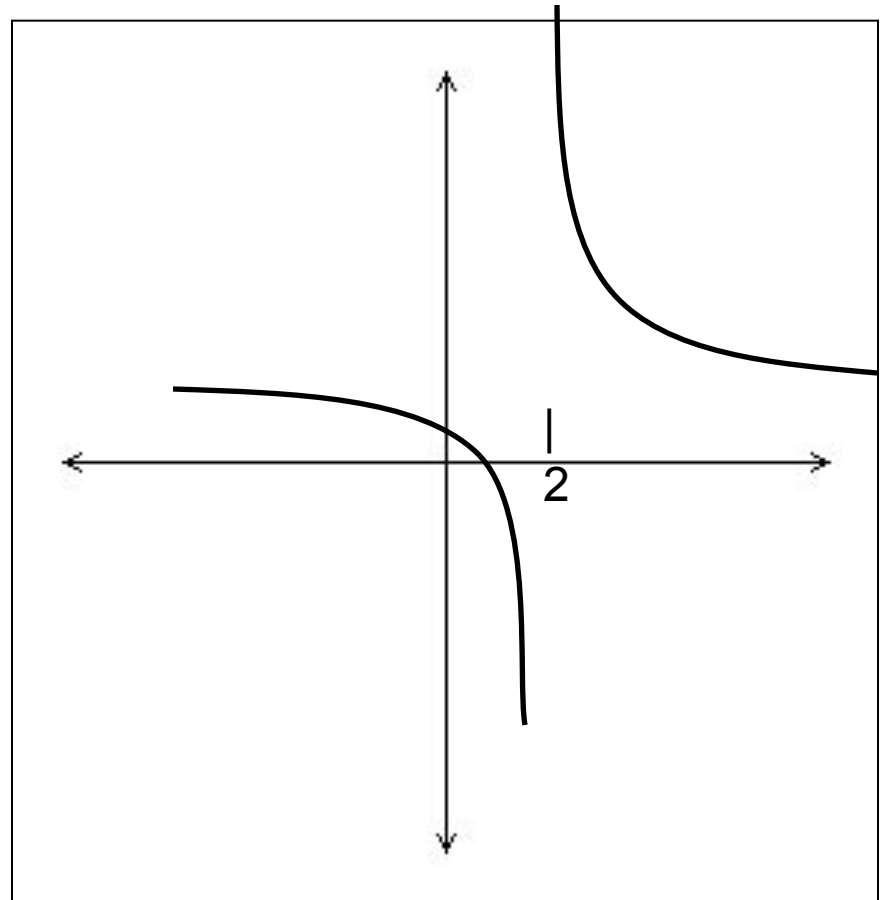


# Nonremovable Discontinuity

(Vertical Asymptote)—Infinite Discontinuity

- Ex:  $f(x) = \frac{x+3}{x-2}$

There is a zero in the denominator of a function that **cannot be “canceled out”** through algebra.



# Finding Discontinuities Summary

- To find vertical asymptotes and holes, factor the problem and simplify.

– If the factor in the bottom cancels out, it gives us a hole.

• Removable  $f(x) = \frac{\cancel{x+2}}{\cancel{(x+2)}(x-1)}$

So the hole occurs at  $x = -2$

How do you find the y-value of the hole?

Simplify and substitute the x-value into the remaining equation!

$$f(-2) = \frac{1}{-2-1}$$

$(-2, -\frac{1}{3})$

– If the factor does not cancel out, it gives us a vertical asymptote.

- Nonremovable

The vertical asymptote for  $f(x)$  is at  $x = 1$

# More Examples:

Find the discontinuities and determine whether they are removable or nonremovable. **Hint: Factor first!**

$$1) f(x) = \frac{x^2 - 4}{x^2 - x - 6}$$

$$2) f(x) = \frac{x^2 - 3x}{x^3 - 7x^2 + 12x}$$

Day 2 - Discontinuity F19

More Examples:  
Find the discontinuities and determine whether they are removable or nonremovable. Hint: Factor first!

1)  $f(x) = \frac{x^2 - 4}{x^2 - x - 6}$

Handwritten notes:  
 $f(x) = \frac{(x-2)(x+2)}{(x+2)(x-3)}$   
 hole at  $x = -2$   
 $y = \frac{-2-2}{-2-3} = \frac{-4}{-5} = \frac{4}{5}$   
 hole at  $(-2, \frac{4}{5})$   
 removable disc.

2)  $f(x) = \frac{x^2 - 3x}{x^3 - 7x^2 + 12x}$

Handwritten notes:  
 $f(x) = \frac{x-2}{x-3}$   
 $x-3=0$   
 V.A. at  $x=3$   
 nonremovable disc.

Day 2 - Discontinuity F19

More Examples:  
Find the discontinuities and determine whether they are removable or nonremovable. Hint: Factor first!

1)  $f(x) = \frac{x^2 - 4}{x^2 - x - 6}$

Handwritten notes:  
 $f(x) = \frac{x(x-3)}{x(x^2 - 7x + 12)}$   
 $f(x) = \frac{x(x-3)}{x(x-3)(x-4)}$   
 holes at  $x=0$  and  $x=3$

2)  $f(x) = \frac{x^2 - 3x}{x^3 - 7x^2 + 12x}$

Handwritten notes:  
 $f(x) = \frac{1}{x-4}$   
 $x=0$   
 $y = \frac{1}{0-4} = -\frac{1}{4}$



## More Examples **ANSWERS:**

Find the discontinuities and determine whether they are removable or nonremovable. **Hint: Factor first!**

$$1) f(x) = \frac{x^2 - 4}{x^2 - x - 6}$$

*Hole*:  $(-2, \frac{4}{5})$

*V.A.*  $x = 3$

$$2) f(x) = \frac{x^2 - 3x}{x^3 - 7x^2 + 12x}$$

*Hole*:  $(0, -\frac{1}{4})$  and  $(3, -1)$

*V.A.*  $x = 4$

# You Try!

Find the discontinuities and determine whether they are removable or nonremovable. **Hint: Factor first!**

$$3) \frac{x+5}{3x^2+13x-10}$$

The screenshot shows a presentation slide titled "Day 2 - Discontinuity F19" with a "You Try!" section. The slide contains handwritten work for problem 3:  $f(x) = \frac{x+5}{3x^2+13x-10}$ . The denominator is factored as  $(3x-2)(x+5)$ , leading to a hole at  $x = -5$  and a vertical asymptote at  $x = \frac{2}{3}$ . The function is simplified to  $y = \frac{1}{3x-2}$ , and a removable discontinuity is identified at  $(-5, -\frac{1}{17})$ . The slide also includes a "You Try!" prompt: "Find the discontinuities and determine whether they are removable or nonremovable. Hint: Factor first!" and lists two other problems: 3)  $\frac{x+5}{3x^2+13x-10}$  and 4)  $\frac{9x^2-81}{3x+9}$ .

$$4) \frac{9x^2-81}{3x+9}$$

The screenshot shows a presentation slide titled "Day 2 - Discontinuity F19" with a "You Try!" section. The slide contains the original problem 4:  $f(x) = \frac{9x^2-81}{3x+9}$ . The slide also includes a "You Try!" prompt: "Find the discontinuities and determine whether they are removable or nonremovable. Hint: Factor first!" and lists two other problems: 3)  $\frac{x+5}{3x^2+13x-10}$  and 4)  $\frac{9x^2-81}{3x+9}$ .

# You Try! ANSWERS

Find the discontinuities and determine whether they are removable or nonremovable. **Hint: Factor first!**

$$\begin{aligned} 3) \quad \frac{x+5}{3x^2+13x-10} &= \frac{\cancel{x+5}}{(\cancel{x+5})(3x-2)} \\ &= \frac{1}{(3x-2)} \end{aligned}$$

$$\text{Hole: } \left(-5, -\frac{1}{17}\right)$$

$$\text{V.A. } x = \frac{2}{3}$$

$$\begin{aligned} 4) \quad \frac{9x^2-81}{3x+9} &= \frac{9\cancel{(x+3)}(x-3)}{3\cancel{(x+3)}} \\ &= 3(x-3) \end{aligned}$$

$$\text{Hole: } (-3, -18)$$

$$\text{V.A. None}$$

## Student Practice:

- A) Classify the function as continuous or discontinuous.
- B) If discontinuous, specify the type of discontinuity and where it exists.
- C) State the domain and x & y intercepts.

1)  $f(x) = (x + 3)(x - 2)$

2)  $f(x) = \frac{x^2 - 4}{x - 2}$

# Student Practice: ANSWERS

- A) Classify the function as continuous or discontinuous.  
B) If discontinuous, specify the type of discontinuity and where it exists.  
C) State the domain and x & y intercepts.

1)  $f(x) = (x + 3)(x - 2)$

*Continuous*

*Domain*:  $(-\infty, \infty)$

*x-int*:  $(-3, 0), (2, 0)$

*y-int*:  $(0, -6)$

2)  $f(x) = \frac{x^2 - 4}{x - 2}$

*Discontinuous – removable (hole)*

Hole at  $(2, 4)$

*Domain*:  $(-\infty, 2) \cup (2, \infty)$

*x-int*:  $(-2, 0)$

*y-int*:  $(0, 2)$

# Try this one!

$$h(x) = \frac{x^3 + x}{x}$$

- Domain:
- x & y intercepts:
- Continuous?
- Type of discontinuity?
- Where is the discontinuity?

The screenshot shows a Beamer presentation slide titled "Day 2 - Discontinuity SC S20". The slide content includes:

- Slide 22: "Try this one!" with the function  $h(x) = \frac{x^3 + x}{x}$  and a list of questions: Domain, x & y intercepts, Continuous?, Type of discontinuity?, and Where is the discontinuity?
- Slide 23: "Try this one! ANSWERS" with the function  $h(x) = \frac{x^3 + x}{x}$  and a list of questions: Domain, x & y intercepts, Continuous?, Type of discontinuity?, and Where is the discontinuity? Handwritten notes in blue ink show the solution:  $x \neq 0$ ,  $(-\infty, 0) \cup (0, \infty)$ ,  $y = x(x^2 + 1)$ ,  $0 = x^2 + 1$ ,  $-1 = x^2$ , and "no x-int.".
- Slide 24: "Let's look at this one together..." with the function  $h(x) = \frac{x^3 + x}{x}$  and a list of questions: Domain, x & y intercepts, Continuous?, Type of discontinuity?, and Where is the discontinuity? Handwritten notes in blue ink show the solution: "Try this one!" and  $h(x) = \frac{x^3 + x}{x}$  with "no y-int." written next to it.

The presentation interface includes a menu bar (File, Edit, View, Insert, Format, Slide, Arrange, Tools, Help), a toolbar (Background, Layout, Theme, Transition), and a footer (add speaker notes).

# Try this one! **ANSWERS**

$$h(x) = \frac{x^3 + x}{x}$$

- Domain: *Domain*:  $(-\infty, 0) \cup (0, \infty)$
- x & y intercepts: *x-int*: *None*      *y-int*: *None*
- Continuous? **No**
- Type of discontinuity? **Removable (Hole)**
- Where is the discontinuity? **Hole at (0, 1)**

# Let's look at this one together...

$$f(x) = \sqrt{16x^4 - 81x^2}$$

- Domain:
- x & y intercepts:
- Continuous?
- Type of discontinuity?

The screenshot shows a Google Slides presentation titled "Day 2 - Discontinuity SC S20". The slide content includes:

- Handwritten equations:  $16x^4 - 81x^2 = 0$ ,  $x^2(16x^2 - 81) = 0$ , and  $x^2(4x+9)(4x-9) = 0$ .
- Handwritten solutions for x:  $x=0, x=-\frac{9}{4}, x=\frac{9}{4}$ .
- Handwritten domain:  $(-\infty, -\frac{9}{4}] \cup [0] \cup [\frac{9}{4}, \infty)$ .
- Handwritten x & y intercepts:  $(0,0), (-\frac{9}{4}, 0), (\frac{9}{4}, 0)$ .
- Handwritten questions: "Continuous?" and "Type of discontinuity?".

The slide also features a number line diagram with points at  $-\frac{9}{4}$ ,  $0$ , and  $\frac{9}{4}$ , and a graph of the function  $f(x) = \sqrt{16x^4 - 81x^2}$ .



# Let's look at this one... ANSWERS

$$f(x) = \sqrt{16x^4 - 81x^2}$$

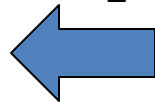
- Domain:  $(-\infty, -\frac{9}{4}] \cup [0] \cup [\frac{9}{4}, \infty)$
- x & y intercepts:  $x\text{-int} : (-\frac{9}{4}, 0), (0, 0), (\frac{9}{4}, 0)$   
 $y\text{-int} : (0, 0)$
- Continuous? **No**
- Type of discontinuity? **None**

Examine the table of values below. All of the following statements are true EXCEPT

$x$	$y_1$	$y_2$
-2.03	-66.67	-4.03
-2.02	-100	-4.02
-2.01	-200	-4.01
-2	ERROR	ERROR
-1.99	200	-3.99
-1.98	100	-3.98
-1.97	66.667	-3.97

$x = -2$

Tomorrow, you'll do an Asymptote Lab to learn more about this.

- A.  $x = -2$  is a vertical asymptote in  $y_1$
- B.  $x = -2$  is an infinite discontinuity in  $y_1$
- C.  $x = -2$  is a removable discontinuity in  $y_2$
- D.  $x = -2$  is a vertical asymptote in  $y_2$  

Looking at  $y_2$ , the  $y$ 's are in order, but  $y = -4$  was skipped, so there is a Removable Discontinuity (Hole) at  $(-2, -4)$

# Definition of Degree

- **Degree of a polynomial in one variable:**  
the value of the greatest exponent

Ex:  $f(x) = 4x^2 + 9x + 8$

**Degree: 2**

Ex:  $g(x) = -5x^3 + 6x^2 + 4x$

**Degree: 3**

- **Degree can help us determine the horizontal asymptote of rational functions...**

# Horizontal Asymptotes

For horizontal asymptotes, think BOSTON for *polynomials*! Looking at the degree of top & bottom...

**B**ottom > **T**op

$$f(x) = \frac{2x}{x^2 + 3x}$$

*H.A. :  $y = 0$*

**y=0**

**S**ame = ratio

$$g(x) = \frac{2x^3}{5x^3 + 4x^2}$$

*H.A. :  $y = \frac{2}{5}$*

**T**op > **B**ottom

**↑** **O** No HA.  
**N**

$$h(x) = \frac{5x^2}{7x + 3}$$

*No H.A.*

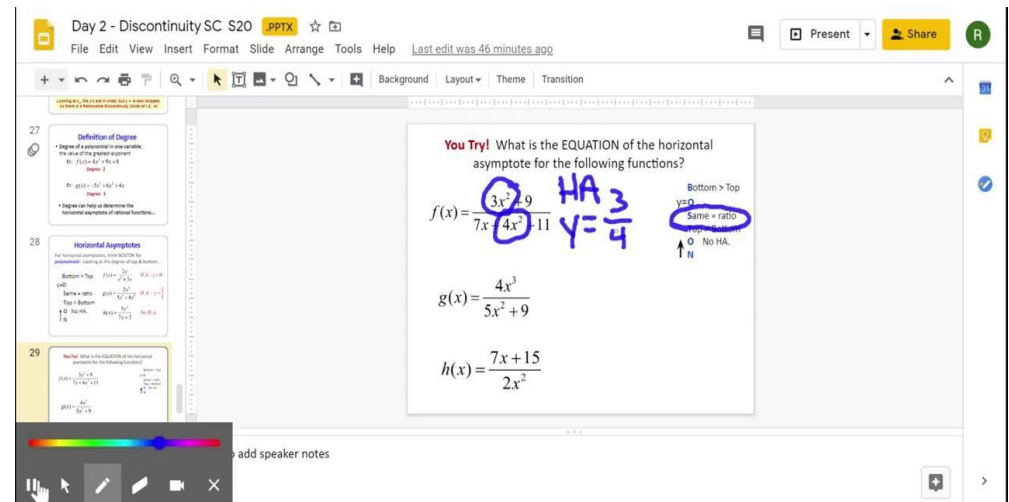
**You Try!** What is the EQUATION of the horizontal asymptote for the following functions?

$$f(x) = \frac{3x^2 + 9}{7x + 4x^2 + 11}$$

Bottom > Top  
 $y=0$   
 Same = ratio  
 Top > Bottom  
 ↑  $0$  No HA.  
 $N$

$$g(x) = \frac{4x^3}{5x^2 + 9}$$

$$h(x) = \frac{7x + 15}{2x^2}$$



**You Try!** What is the EQUATION of the horizontal asymptote for the following functions?

$$f(x) = \frac{3x^2 + 9}{7x + 4x^2 + 11}$$

$$H.A. : y = \frac{3}{4}$$

Bottom > Top  
y=0  
Same = ratio  
Top > Bottom  
↑ 0 No HA.  
N

$$g(x) = \frac{4x^3}{5x^2 + 9}$$

$$H.A. : \text{none}$$

$$h(x) = \frac{7x + 15}{2x^2}$$

$$H.A. : y = 0$$

# You Try: True or False

- 1) The graph of function  $f$  is defined as the set of all points  $(x, f(x))$  where  $x$  is in the domain of  $f$ . Justify your answer.
- 2) If a function is not continuous, then the domain cannot be all real numbers.

# True or False    ANSWERS

- 1) The graph of function  $f$  is defined as the set of all points  $(x, f(x))$  where  $x$  is in the domain of  $f$ . Justify your answer.

**True! This is the definition of a function.**

- 2) If a function is not continuous, then the domain cannot be all real numbers.

**False! It could be a piecewise function.**



# Is this continuous?

State whether the scenario is continuous or discontinuous.

- A) Outdoor temperature as a function of time.

**Continuous**

- B) Number of soft drinks sold at a ballpark as a function of outdoor temp. **Discontinuous**

- C) Your hair length as a function of days in a year

**Continuous**

# Homework Day 2

- Packet p. 2-3