Notes Day 1

NEW with some notes written in AND more vocab!

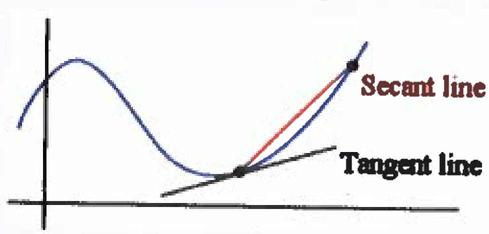
Spr '18

Introduction to Derivatives Webquest

- Answer the questions on the handout.
- Take Notes on other key points! ☺
- Be prepared to discuss afterwards.

Secant vs Tangent

- Tangent lines touch a curve at one point
- Slope at that one point is *instantaneous* rate of change.
- Secant lines cut through a curve at two points
- The slope of a secant line between those two points it is called the *average* rate of change.

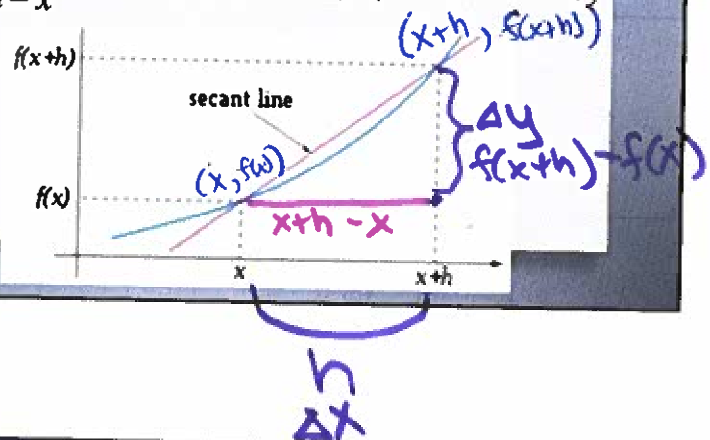


Slope – all the same!

$$m = rate\ of\ change = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{change in y}}{\text{change in x}} = \frac{\Delta y}{\Delta x}$$

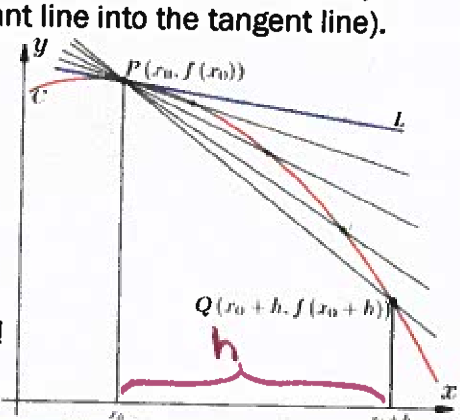
$$m = \frac{f(x_2) - f(x_1)}{x_2 - x_1} \quad where: (x_1, f(x_1)); (x_2, f(x_2))$$

$$\Rightarrow m = \frac{f(x+h) - f(x)}{x+h-x} \quad where: (x, f(x)); (x+h, f(x+h))$$



Use Secant or Tangent?

- Tangent lines are REALLY hard to draw. So you can draw a secant line and calculate its slope as one point on the line gets closer and closer to the point of tangency (thus, making the secant line into the tangent line).
- Of course, the change in x (the h here) would be 0 if the two points actually made it on top of each other.
- That's where the idea of limits comes in here!



Limit Definition of Derivative!

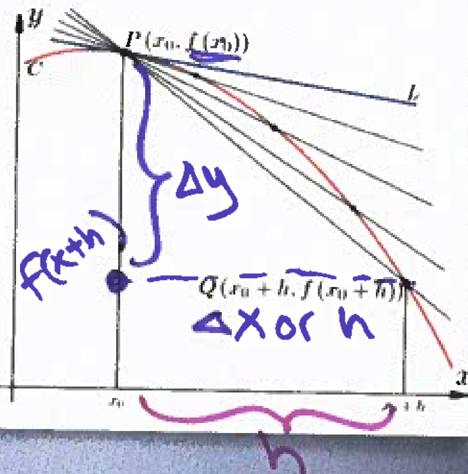
Building off of the last slope you wrote down

$$m = \frac{f(x+h) - f(x)}{x+h-x}$$

The Limit Definition of Derivative is

$$f'(x) = \frac{d}{dx} f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$f' =$



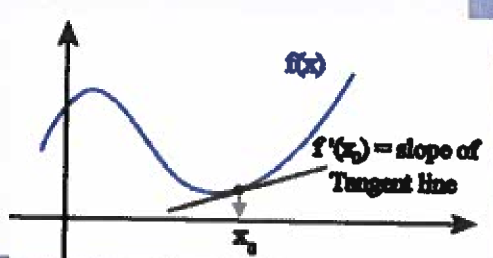
Limit definition of derivative

The derivative with respect to x is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

The derivative is

- The slope of the tangent line at a single point on the curve.
- The Instantaneous rate of change



What do we need to write an equation of a line?

1) One point and slope; then use point slope formula $y - y_1 = m(x - x_1)$
2) Two points; then compute the slope and use one of the points in the point slope formula

→ We'll do this at a later date!!

Notation of Derivative

- Given f(x) = 3x² + 5x.

$$\boxed{\frac{dy}{dx},\ y',\ f'(x)}$$

We could say…

- $f'(x) = 6x + 5$ or
- $y' = 6x + 5$ or
- $\frac{dy}{dx} = 6x + 5$

f' or y' is called "f prime or y prime"

$\frac{dy}{dx}$ is said "derivative of y with respect to x"

x is variable

Example 1:

Evaluate the derivative using the limit definition of derivatives. $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

- Function: $f(x)=4x+9$

$f' = \lim_{h\to 0}\frac{(4(x+h)+9)-(4x+9)}{h}$

$= \lim_{h\to 0}\frac{(4x+4h+9)-4x-9}{h} = \boxed{4}$

$= \lim_{h\to 0}\frac{4\cancel{h}}{\cancel{h}} = \boxed{4}$

Example 2:

Evaluate the derivative using the limit definition of derivatives. $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

- Function: $f(x) = x^2 + 2x + 3$

$$f' = \lim_{h \to 0} \frac{\left((x+h)^2 + 2(x+h) + 3\right) - (x^2 + 2x + 3)}{h}$$

$$= \lim_{h \to 0} \frac{(\cancel{x^2} + 2xh + h^2 + \cancel{2x} + 2h + \cancel{3}) - \cancel{x^2} - \cancel{2x} - \cancel{3}}{h}$$

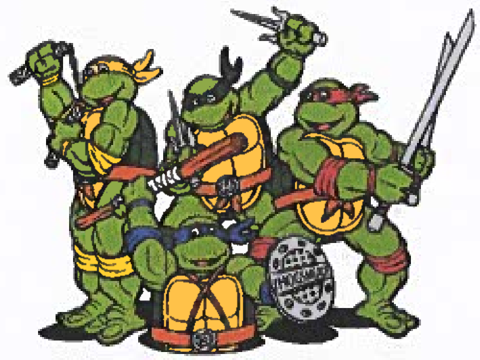
$$\boxed{= 2x + 2}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 + 2h}{h} = \lim_{h \to 0} \frac{\cancel{h}(2x + h + 2)}{\cancel{h}}$$

$$= \lim_{h \to 0} 2x + h + 2 = \boxed{2x + 2}$$

Classwork: Packet p.2

- What did the ninja turtles say when handed the expression....?



HW = Packet p 19
Spr '18

= practice with function notation and part of the limit definition problems we studied today