Unit 2 Day 1
MATRICES

MATRIX OPERATIONS

Warm-Up
Basic Matrix Practice

1. $\begin{bmatrix} 4 & -a \\ 6 & -3 \end{bmatrix} + \begin{bmatrix} -6 & -5 \\ 7a & 3 \end{bmatrix}$

2. $5\begin{bmatrix} 1 & -2 & x \\ 4 & y & 1 \\ 0 & -5 & x^2 \end{bmatrix}$

3. $\begin{bmatrix} 1 & 3 & -2 \\ 4 & 0 & 5 \end{bmatrix} - \begin{bmatrix} 2 & -1 & 5b \\ 6c & 4 & -3 \end{bmatrix}$

4. $3\begin{bmatrix} -3t & 0 \\ 4 & 5q \end{bmatrix}$

Warm-Up ANSWERS!

Matrix Addition & Subtraction:

1. $$\begin{bmatrix} 4 & -a \\ 6 & -3 \end{bmatrix} + \begin{bmatrix} -6 & -5 \\ 7a & 3 \end{bmatrix} = \begin{bmatrix} -2 & -a-5 \\ 7a+6 & 0 \end{bmatrix}$$

3. $$\begin{bmatrix} 1 & 3 & -2 \\ 4 & 0 & 5 \end{bmatrix} - \begin{bmatrix} 2 & -1 & 5b \\ 6c & 4 & -3 \end{bmatrix} = \begin{bmatrix} -1 & 4 & -2-5b \\ 4-6c & -4 & 8 \end{bmatrix}$$

Warm-Up ANSWERS!

Scalar Multiplication:

2. $$5\begin{bmatrix} 1 & -2 & x \\ 4 & y & 1 \\ 0 & -5 & x^2 \end{bmatrix} = \begin{bmatrix} 5 & -10 & 5x \\ 20 & 5y & 5 \\ 0 & -25 & 5x^2 \end{bmatrix}$$

4. $$3\begin{bmatrix} -3t & 0 \\ 4 & 5q \end{bmatrix} = \begin{bmatrix} -9t & 0 \\ 12 & 15q \end{bmatrix}$$

Questions About HW?

Questions About Matrices Basics HW?

Tonight's HW

Is Packet p. 1-2

A heads-up…
The HW is not in order in the packet, so be sure to refer to your outline this unit. ☺

Unit 2 Day 1
NOTES Part 1

BASIC MATRIX OPERATIONS

& APPLICATIONS

Remember…

Matrix: a rectangular array of numbers or variables used to organize data.

$$A = \begin{pmatrix} 2 & -4 \\ -3 & 5 \\ 1 & 6 \end{pmatrix}$$

Typically, we name matrices with capital letters!

The numbers in the matrix are called elements. There are 6 elements in this matrix.

Remember…

Matrix dimensions tell how many ROWS & COLUMNS there are in the matrix.

Dimensions & Notation

- $A_{3 \times 2}$
- **REMEMBER:**
  RC cola
  Rows by columns

$$A = \begin{pmatrix} 2 & -4 \\ -3 & 5 \\ 1 & 6 \end{pmatrix}$$

Rows run horizontally & columns run vertically.

The dimensions of two matrices can determine whether or not they may be added, subtracted, or multiplied.

A matrix of *m* rows and *n* columns is called a matrix with **dimensions *m* x *n*.**

You Try Examples: Find the dimensions.

1.) $\begin{bmatrix} 2 & -3 & 4 \\ -1 & \frac{1}{2} & \pi \end{bmatrix}$

2 X 3

2.) $\begin{bmatrix} -3 & 8 & 9 \\ \pi & -2 & 5 \\ -6 & 7 & 8 \end{bmatrix}$

3 X 3

3.) $\begin{bmatrix} 10 \\ -7 \end{bmatrix}$ **2 X 1**

4.) $\begin{bmatrix} -3 & 4 \end{bmatrix}$

1 X 2

Matrix Position

We can give the location of an element in a matrix by naming its row, then its column.

$$\begin{pmatrix} 2 & -4 & -8 & 4 \\ -3 & 5 & -6 & 3 \\ 1 & 6 & -2 & 0 \end{pmatrix}$$

3 is in position k_{24}

6 is in position k_{32}

4 is in position k_{14}

-2 is in position k_{33}

Organizing Data Into Matrices

Identify each matrix element.

$$K = \begin{bmatrix} 3 & -1 & -8 & 5 \\ 1 & 8 & 4 & 9 \\ 8 & -4 & 7 & -5 \end{bmatrix}$$

a. k_{12} **b.** k_{32} **c.** k_{23} **d.** k_{34}

a. $K = \begin{bmatrix} 3 & -1 & -8 & 5 \\ 1 & 8 & 4 & 9 \\ 8 & -4 & 7 & -5 \end{bmatrix}$

k_{12} is the element in the first row and second column.

Element k_{12} is -1.

b. $K = \begin{bmatrix} 3 & -1 & -8 & 5 \\ 1 & 8 & 4 & 9 \\ 8 & -4 & 7 & -5 \end{bmatrix}$

k_{32} is the element in the third row and second column.

Element k_{32} is -4.

Why use matrices?

- Matrix algebra makes *mathematical expression and computation* easier.
- It allows you to get rid of cumbersome notation, concentrate on the concepts involved and understand where your results come from.
- Matrices are used to represent real-world data such as the *habits* or *traits of populations.*

Special Matrices

Some matrices have special names because of what they look like.

a) **<u>Row matrix</u>:** only has 1 row. $Ex)\ \begin{bmatrix} -3 & 4 \end{bmatrix}$

b) **<u>Column matrix</u>:** only has 1 column. $Ex)\ \begin{bmatrix} 10 \\ -7 \end{bmatrix}$

c) **<u>Square matrix</u>:** has the same number of rows and columns. $Ex)\ \begin{bmatrix} 10 & -2 \\ -7 & 3 \end{bmatrix}$

d) **<u>Zero matrix</u>:** contains all zeros.

$$Ex)\ \begin{bmatrix} 0 & 0 \end{bmatrix}$$

Remember...
Matrix Addition & Subtraction

- You can add or subtract matrices if they have the **same dimensions** (same number of rows and columns).
- To do this, you add (or subtract) the corresponding numbers (numbers in the same positions).
- If a matrix operation is not possible for a problem, the solution is called **undefined**.

Ex:

$$\begin{bmatrix} 2 & -4 \\ 5 & 0 \\ 1 & -3 \end{bmatrix} - \begin{bmatrix} -1 & -2 & 3 \\ 0 & 1 & -3 \end{bmatrix} = \quad \textbf{\textit{Undefined}}$$

Properties of Matrix Addition

- Matrix addition **IS** commutative

$A + B = B + A$

Order does NOT Matter!

- Matrix addition **IS** associative

$A + (B + C) = (A + B) + C$

Grouping does NOT Matter!

Remember...
Scalar Multiplication

- To do this, multiply each entry in the matrix by the number outside (called the scalar).

  This is like distributing a number to a polynomial.

Example:

$$4\begin{bmatrix} 2 & -4 \\ 5 & 0 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} 8 & -16 \\ 20 & 0 \\ 4 & -12 \end{bmatrix}$$

Unit 2 Day 1
NOTES Part 2

MATRIX MULTIPLICATION

Matrix Multiplication

- Matrix Multiplication is **NOT Commutative**! Order matters!
- You can multiply matrices only if the number of columns in the first matrix equals the number of rows in the second matrix.

2 columns

$$\begin{bmatrix} 2 & 3 \\ -5 & 6 \\ 9 & -7 \end{bmatrix} \bullet \begin{bmatrix} 1 & -2 & 0 \\ 3 & 4 & -5 \end{bmatrix}$$

2 rows

3 x 2 2 x 3 **= 3 x 3 resulting matrix**

Matrix Multiplication

- Take the numbers in the first row of matrix #1. Multiply each number by its corresponding number in the first column of matrix #2. Total these products.

$$\begin{bmatrix} 2 & 3 \\ -5 & 6 \\ 9 & -7 \end{bmatrix} \bullet \begin{bmatrix} 1 & -2 & 0 \\ 3 & 4 & -5 \end{bmatrix}$$

$$2 \bullet 1 + 3 \bullet 3 = 11$$

Do 1st • 1st + 2nd • 2nd +...

The result, 11, goes in row 1, column 1 of the answer. Repeat with row 1, column 2; row 1 column 3; row 2, column 1; ...

Continued on the next slide....

Let's Try!

$$\begin{bmatrix} 2 & 3 \\ -5 & 6 \\ 9 & -7 \end{bmatrix} \bullet \begin{bmatrix} 1 & -2 & 0 \\ 3 & 4 & -5 \end{bmatrix}$$

Let's Try Answer!

$$\begin{bmatrix} 2 & 3 \\ -5 & 6 \\ 9 & -7 \end{bmatrix} \bullet \begin{bmatrix} 1 & -2 & 0 \\ 3 & 4 & -5 \end{bmatrix} = \begin{bmatrix} 11 & 8 & -15 \\ 13 & 34 & -30 \\ -12 & -46 & 35 \end{bmatrix}$$

You Try!

$$\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 3 & -9 & 2 \\ 5 & 7 & -6 \end{bmatrix}$$

$$\begin{bmatrix} & & \\ & & \end{bmatrix}$$

You Try! Answer

$$\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 3 & -9 & 2 \\ 5 & 7 & -6 \end{bmatrix}$$

2 x 2 2 x 3

$$\begin{bmatrix} 2(3) + -1(5) & 2(-9) + -1(7) & 2(2) + -1(-6) \\ 3(3) + 4(5) & 3(-9) + 4(7) & 3(2) + 4(-6) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -25 & 10 \\ 29 & 1 & -18 \end{bmatrix}$$

Matrix Multiplication

Find AB and BA given $A = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ -9 & 0 \\ 10 & -5 \end{bmatrix}$

AB is *undefined*. A 2 x 1 and 3 x 2 cannot be multiplied.

$$BA = \begin{bmatrix} 2 & 1 \\ -9 & 0 \\ 10 & -5 \end{bmatrix} \bullet \begin{bmatrix} 5 \\ -2 \end{bmatrix} = \begin{bmatrix} 8 \\ -45 \\ 60 \end{bmatrix}$$

Matrix Multiplication Properties

- Matrix multiplication is **NOT** commutative

  $AB \neq BA$

- Matrix multiplication **IS** associative

  $A(BC)=(AB)C$

- Matrix multiplication **IS** distributive

  $A(B+C)=AB+AC$

  $(A+B)C=AC+BC$

Tip for Applications of Matrix Multiplication

Remember from before:

- Matrix Multiplication is **NOT Commutative**! Order matters!
- You can multiply matrices only if the number of columns in the first matrix equals the number of rows in the second matrix.

(3 x 2) • (2 x 3) = **3 x 3 matrix**

Therefore, we **must be strategic** when setting up an application problem – and **match up "inner dimensions"**!

Packet p. 2 #1
Matrix Applications

Two softball teams submit equipment lists for the season.
Women's Team: 12 bats, 45 balls, 15 uniforms

Men's Team: 15 bats, 38 balls 17 uniforms

Each bat costs $21, each ball costs $4, and each uniform costs $30.

Use matrix multiplication to find the total cost of equipment for each team.

Application Answer

EQUIPMENT

	Bats	Balls	Uniforms
Women's team	12	45	15
Men's team	15	38	17

COST

	Dollars
Bats	21
Balls	4
Uniforms	30

The total cost of equipment for each team can now be obtained by multiplying the equipment matrix by the cost per item matrix. The equipment matrix is 2×3 and the cost per item matrix is 3×1, so their product is a 2×1 matrix.

$$\begin{bmatrix} 12 & 45 & 15 \\ 15 & 38 & 17 \end{bmatrix} \begin{bmatrix} 21 \\ 4 \\ 30 \end{bmatrix} = \begin{bmatrix} 12(21) + 45(4) + 15(30) \\ 15(21) + 38(4) + 17(30) \end{bmatrix} = \begin{bmatrix} 882 \\ 977 \end{bmatrix}$$

The labels for the product matrix are as follows.

TOTAL COST

	Dollars
Women's team	882
Men's team	977

Tip: Create a "boomerang" with the "like" info!

- The total cost of equipment for the women's team is \$882, and the total cost of equipment for the men's team is \$977.

Practice on Packet p. 2

= Matrix Applications #2 and 3

Homework

- Packet p. 1 and 2

Matrix Addition Examples

Ex 1:

$$\begin{bmatrix} 2 & -4 \\ 5 & 0 \\ 1 & -3 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ -2 & 1 \\ 3 & -3 \end{bmatrix} =$$

Ex 2:

$$\begin{bmatrix} 2 & -4 \\ 5 & 0 \\ 1 & -3 \end{bmatrix} - \begin{bmatrix} -1 & -2 & 3 \\ 0 & 1 & -3 \end{bmatrix} =$$

Matrix Addition Example ANSWERS

Ex 1:

$$\begin{bmatrix} 2 & -4 \\ 5 & 0 \\ 1 & -3 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ -2 & 1 \\ 3 & -3 \end{bmatrix} =$$

$$\begin{bmatrix} 3 & -4 \\ 7 & -1 \\ -2 & 0 \end{bmatrix}$$

Ex 2:

$$\begin{bmatrix} 2 & -4 \\ 5 & 0 \\ 1 & -3 \end{bmatrix} - \begin{bmatrix} -1 & -2 & 3 \\ 0 & 1 & -3 \end{bmatrix} =$$

Undefined

Adding and Subtracting Matrices

1 EXAMPLE The table shows information on ticket sales for a new movie that is showing at two theaters. Sales are for children (*C*) and adults (*A*).

Theater	*C*	*A*	*C*	*A*
1	198	350	54	439
2	201	375	58	386

a. Write two 2×2 matrices to represent matinee and evening sales.

b. Find the combined sales for the two showings.

ANSWERS Adding and Subtracting Matrices

1 EXAMPLE The table shows information on ticket sales for a new movie that is showing at two theaters. Sales are for children (*C*) and adults (*A*).

Theater	*C*	*A*	*C*	*A*
1	198	350	54	439
2	201	375	58	386

a. Write two 2 × 2 matrices to represent matinee and evening sales.

Matinee

$$\begin{array}{l} \\ \text{Theater 1} \\ \text{Theater 2} \end{array} \begin{array}{c} \begin{array}{cc} C & A \end{array} \\ \begin{bmatrix} 198 & 350 \\ 201 & 375 \end{bmatrix} \end{array}$$

Evening

$$\begin{array}{l} \\ \text{Theater 1} \\ \text{Theater 2} \end{array} \begin{array}{c} \begin{array}{cc} C & A \end{array} \\ \begin{bmatrix} 54 & 439 \\ 58 & 386 \end{bmatrix} \end{array}$$

ANSWERS Adding and Subtracting Matrices

EXAMPLE 1 **(continued)**

b. Find the combined sales for the two showings.

$$\begin{bmatrix} 198 & 350 \\ 201 & 375 \end{bmatrix} + \begin{bmatrix} 54 & 439 \\ 58 & 386 \end{bmatrix} = \begin{bmatrix} 198 + 54 & 350 + 439 \\ 201 + 58 & 375 + 386 \end{bmatrix}$$

$$= \begin{array}{l} \\ \text{Theater 1} \\ \text{Theater 2} \end{array} \begin{array}{c} \begin{array}{cc} C & A \end{array} \\ \begin{bmatrix} 252 & 789 \\ 259 & 761 \end{bmatrix} \end{array}$$

Adding & Subtracting Matrices

You can perform matrix addition on matrices with equal dimensions.

a. $\begin{bmatrix} 9 & 0 \\ -4 & 6 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 9+0 & 0+0 \\ -4+0 & 6+0 \end{bmatrix}$

$= \begin{bmatrix} 9 & 0 \\ -4 & 6 \end{bmatrix}$

b. $\begin{bmatrix} 3 & -8 \\ -5 & 1 \end{bmatrix} + \begin{bmatrix} -3 & 8 \\ 5 & -1 \end{bmatrix}$

$= \begin{bmatrix} 3+(-3) & -8+8 \\ -5+5 & 1+(-1) \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$