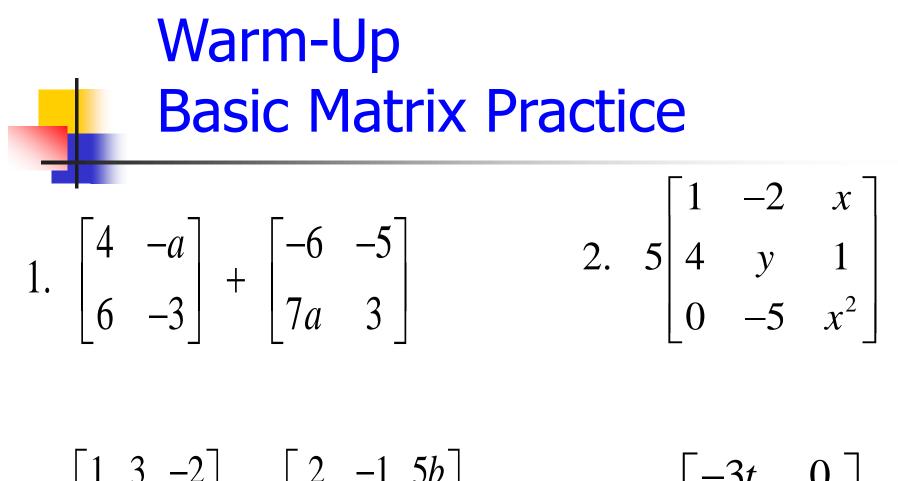
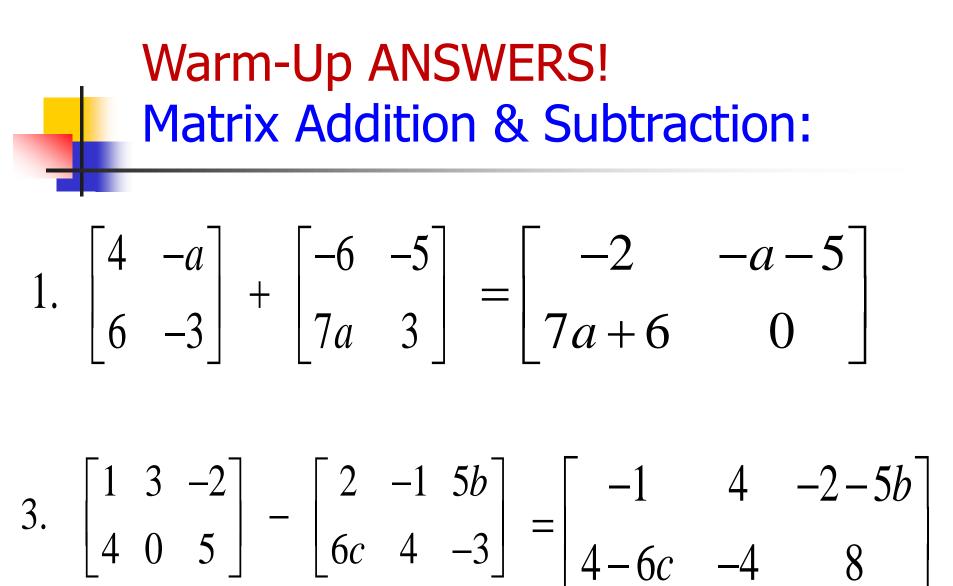
Unit 2 Day 1 MATRICES

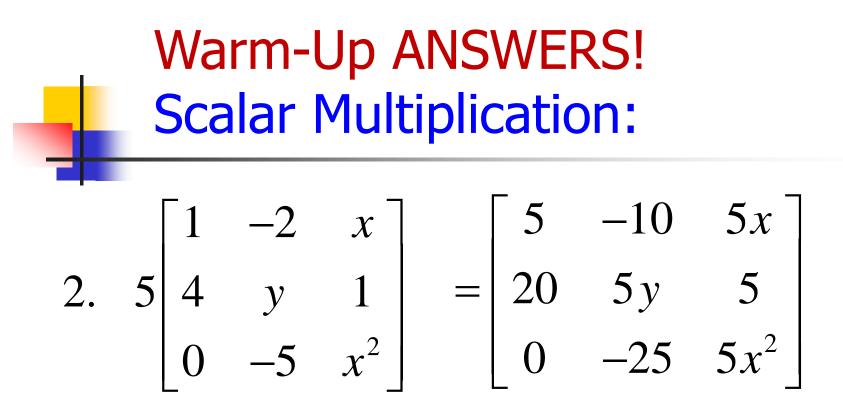
MATRIX OPERATIONS



3.
$$\begin{bmatrix} 1 & 3 & -2 \\ 4 & 0 & 5 \end{bmatrix} - \begin{bmatrix} 2 & -1 & 5b \\ 6c & 4 & -3 \end{bmatrix}$$

 $4. \quad 3 \begin{vmatrix} -3t & 0 \\ 4 & 5q \end{vmatrix}$





4. $3\begin{bmatrix} -3t & 0\\ 4 & 5q \end{bmatrix} = \begin{bmatrix} -9t & 0\\ 12 & 15q \end{bmatrix}$



Questions About Matrices Basics HW?



Is Packet p. 1-2

A heads-up... The HW is not in order in the packet, so be sure to refer to your outline this unit. ③ Unit 2 Day 1 NOTES Part 1

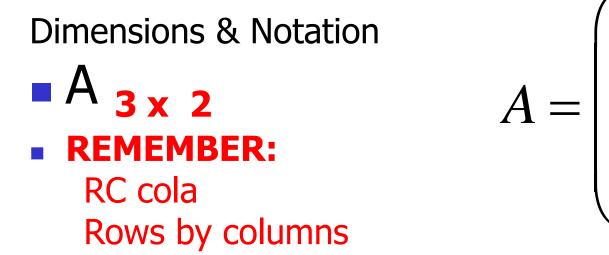
BASIC MATRIX OPERATIONS & APPLICATIONS

Remember... Matrix: a rectangular array of numbers or variables used to organize data.

$$A = \begin{pmatrix} 2 & -4 \\ -3 & 5 \\ 1 & 6 \end{pmatrix}$$

Typically, we name matrices with capital letters!

The numbers in the matrix are called elements. There are 6 elements in this matrix. Remember... Matrix dimensions tell how many ROWS & COLUMNS there are in the matrix.



 $A = \begin{pmatrix} 2 & -4 \\ -3 & 5 \\ 1 & 6 \end{pmatrix}$

Rows run horizontally & columns run vertically.

The dimensions of two matrices can determine whether or not they may be added, subtracted, or multiplied. A matrix of m rows and n columns is called a matrix with **dimensions** $m \ge n$.

 You Try Examples: Find the dimensions.

 1.)
 $\begin{bmatrix} 2 & -3 & 4 \\ -1 & \frac{1}{2} & \pi \end{bmatrix}$ 2.)
 $\begin{bmatrix} -3 & 8 & 9 \\ \pi & -2 & 5 \\ -6 & 7 & 8 \end{bmatrix}$

 2 X 3
 3 X 3

4.)[-3 4]

1 X 2

3.) $\begin{bmatrix} 10 \\ -7 \end{bmatrix}$ **2 X 1**

Matrix Position

We can give the location of an element in a matrix by naming its row, then its column.

$$\begin{pmatrix} 2 & -4 & -8 & 4 \\ -3 & 5 & -6 & 3 \\ 1 & 6 & -2 & 0 \end{pmatrix}$$

3 is in position
$$\frac{k_{24}}{k_{32}}$$

6 is in position $\frac{k_{32}}{k_{14}}$
4 is in position $\frac{k_{14}}{k_{33}}$

Organizing Data Into Matrices

Identify each matrix element.

$$K = \begin{bmatrix} 3 & -1 & -8 & 5 \\ 1 & 8 & 4 & 9 \\ 8 & -4 & 7 & -5 \end{bmatrix}$$
a. k_{12}
b. k_{32}
a. $K = \begin{bmatrix} 3 & -1 & -8 & 5 \\ 1 & 8 & 4 & 9 \\ 8 & -4 & 7 & -5 \end{bmatrix}$

 k_{12} is the element in the first row and second column.

Element k_{12} is -1.

c.
$$k_{23}$$
 d. k_{34}
b. $K = \begin{bmatrix} 3 & -1 & -8 & 5 \\ 1 & 8 & 4 & 9 \\ 8 & -4 & 7 & -5 \end{bmatrix}$

 k_{32} is the element in the third row and second column.

Element k_{32} is -4.

Why use matrices?

- Matrix algebra makes *mathematical expression* and computation easier.
- It allows you to get rid of cumbersome notation, concentrate on the concepts involved and understand where your results come from.
- Matrices are used to represent real-world data such as the *habits* or *traits of populations*.

Special Matrices Some matrices have special names because of what they look like.

- **a) Row matrix**: only has 1 row. Ex) $\begin{bmatrix} -3 & 4 \end{bmatrix}$
- **b)** Column matrix: only has 1 column. Ex $\begin{vmatrix} 10 \\ -7 \end{vmatrix}$
- **c)** Square matrix: has the same number of rows and columns. **b**(10) -2Ex) $\begin{bmatrix} 10 & -2 \\ -7 & 3 \end{bmatrix}$
- **d) Zero matrix**: contains all zeros. Ex) $\begin{bmatrix} 0 & 0 \end{bmatrix}$

Remember... Matrix Addition & Subtraction

- You can add or subtract matrices if they have the same dimensions (same number of rows and columns).
- To do this, you add (or subtract) the corresponding numbers (numbers in the same positions).
- If a matrix operation is not possible for a problem, the solution is called undefined.

Ex:

$$\begin{bmatrix} 2 & -4 \\ 5 & 0 \\ 1 & -3 \end{bmatrix} - \begin{bmatrix} -1 & -2 & 3 \\ 0 & 1 & -3 \end{bmatrix} =$$
Undefine

Properties of Matrix Addition

Matrix addition IS <u>commutative</u>

A + B = B + A Order does NOT Matter!

• Matrix addition **IS** <u>associative</u> A + (B + C) = (A + B) + C



Remember... Scalar Multiplication

- To do this, multiply each entry in the matrix by the number outside (called the scalar).
 - This is like distributing a number to a polynomial.

Unit 2 Day 1 NOTES Part 2

MATRIX MULTIPLICATION

Matrix Multiplication

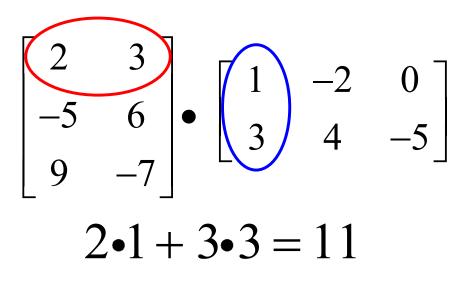
- Matrix Multiplication is <u>NOT Commutative</u>! Order matters!
- You can multiply matrices <u>only</u> if the number of columns in the first matrix equals the number of rows in the second matrix.

2 columns
$$\begin{bmatrix} 2 & 3 \\ -5 & 6 \\ 9 & -7 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 & 0 \\ 3 & 4 & -5 \end{bmatrix} \leftarrow 2 \text{ rows}$$

3 x 2 2 x 3 = 3 x 3
resulting matrix

Matrix Multiplication

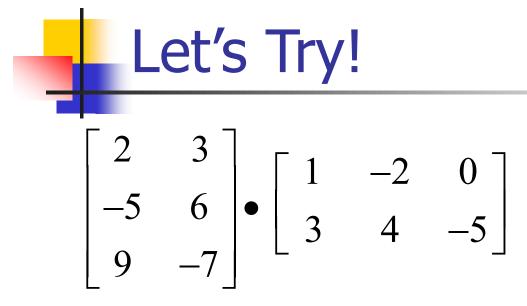
 Take the numbers in the first row of matrix #1. Multiply each number by its corresponding number in the first column of matrix #2. Total these products.



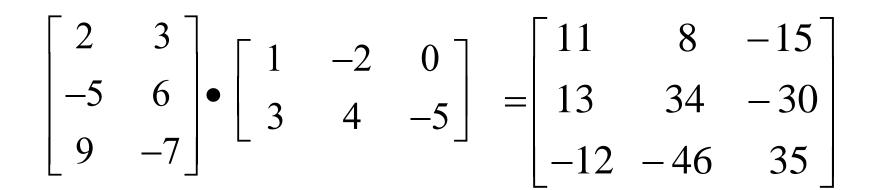
Do 1st • 1st + 2nd • 2nd +...

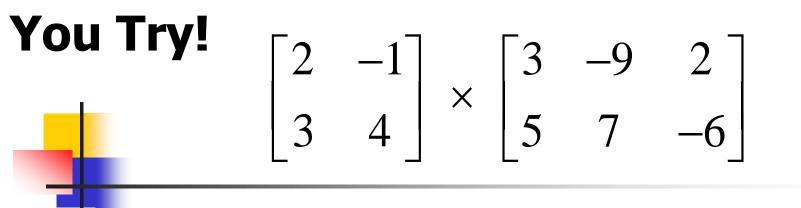
 $\begin{bmatrix} 1 \\ 3 \end{bmatrix} \begin{bmatrix} -2 & 0 \\ 4 & -5 \end{bmatrix}$ The result, 11, goes in row 1, column 1 of the answer. Repeat with row 1, column 2; row 1 column 3; row 2, column 1; ...

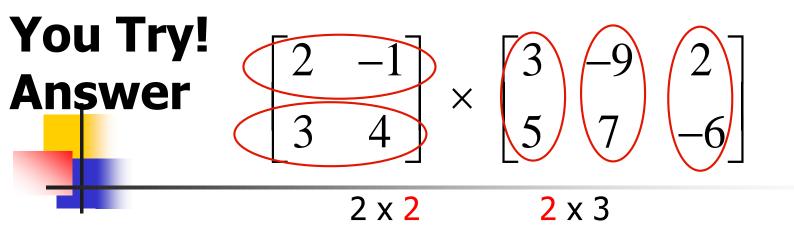
Continued on the next slide....



Let's Try Answer!







2(3) + -1(5) 2(-9) + -1(7) 2(2) + -1(-6)3(3) + 4(5) 3(-9) + 4(7) 3(2) + 4(-6)

-25 10 1 -18

Matrix MultiplicationFind AB and BA given $A = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ -9 & 0 \\ 10 & -5 \end{bmatrix}$

AB is *undefined*. A 2 x 1 and 3 x 2 cannot be multiplied.

$$BA = \begin{bmatrix} 2 & 1 \\ -9 & 0 \\ 10 & -5 \end{bmatrix} \bullet \begin{bmatrix} 5 \\ -2 \end{bmatrix} = \begin{bmatrix} 8 \\ -45 \\ 60 \end{bmatrix}$$

Matrix Multiplication Properties

- Matrix multiplication is NOT <u>commutative</u> AB≠BA
- Matrix multiplication IS <u>associative</u> A(BC)=(AB)C
- Matrix multiplication IS <u>distributive</u>

A(B+C)=AB+AC(A+B)C=AC+BC

Tip for Applications of Matrix Multiplication

Remember from before:

 Matrix Multiplication is <u>NOT Commutative</u>! Order matters!

You can multiply matrices <u>only</u> if the number of columns in the first matrix equals the number of rows in the second matrix.
 (3 x 2) • (2 x 3) = 3 x 3 matrix

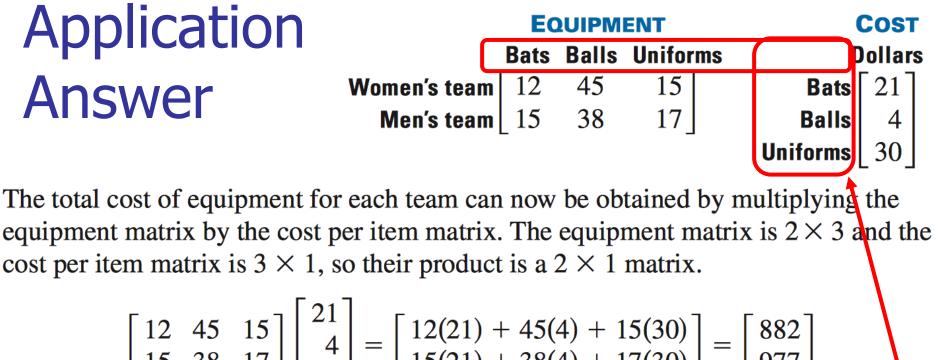
Therefore, we **must be strategic** when setting up an application problem – and **match up "inner dimensions**"!

Packet p. 2 #1 Matrix Applications

Two softball teams submit equipment lists for the season. **Women's Team**: 12 bats, 45 balls, 15 uniforms **Men's Team**: 15 bats, 38 balls 17 uniforms

Each bat costs \$21, each ball costs \$4, and each uniform costs \$30.

Use matrix multiplication to find the total cost of equipment for each team.



$$\begin{bmatrix} 12 & 45 & 15 \\ 15 & 38 & 17 \end{bmatrix} \begin{bmatrix} -1 \\ 4 \\ 30 \end{bmatrix} = \begin{bmatrix} 12(21) + 45(4) + 15(30) \\ 15(21) + 38(4) + 17(30) \end{bmatrix} = \begin{bmatrix} 882 \\ 977 \end{bmatrix}$$

The labels for the product matrix are as follows.

TOTAL COST

Dollars Women's team 882 Men's team 977

Tip: Create a "boomerang" with the "like" info!

The total cost of equipment for the women's team is \$882, and the total cost of equipment for the men's team is \$977.

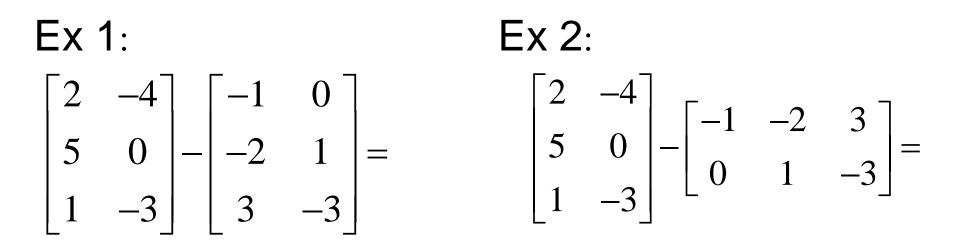
Practice on Packet p. 2

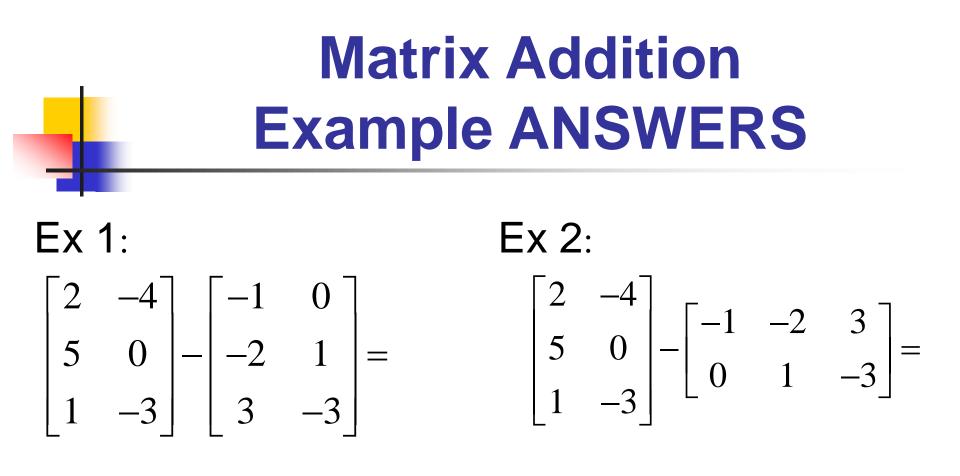
= Matrix Applications #2 and 3



Packet p. 1 and 2

Matrix Addition Examples





 $\begin{bmatrix} 3 & -4 \\ 7 & -1 \\ -2 & 0 \end{bmatrix}$

Undefined

Adding and Subtracting Matrices

EXAMPLE The table shows information on ticket sales for a new movie that is showing at two theaters. Sales are for children (C) and adults (A).

Theater	С	A	С	Α
1	198	350	54	439
2	201	375	58	386

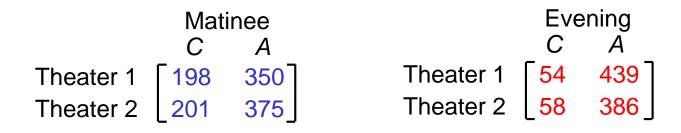
- **a.** Write two 2×2 matrices to represent matinee and evening sales.
- **b.** Find the combined sales for the two showings.

ANSWERS Adding and Subtracting Matrices

EXAMPLE The table shows information on ticket sales for a new movie that is showing at two theaters. Sales are for children (C) and adults (A).

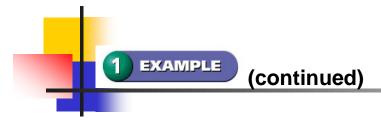
<u> </u>	A	С	A
198	350	54	439
201	375	58	386

a. Write two 2×2 matrices to represent matinee and evening sales.





ANSWERS Adding and Subtracting Matrices



b. Find the combined sales for the two showings.

$$\begin{bmatrix} 198 & 350 \\ 201 & 375 \end{bmatrix} + \begin{bmatrix} 54 & 439 \\ 58 & 386 \end{bmatrix} = \begin{bmatrix} 198 + 54 & 350 + 439 \\ 201 + 58 & 375 + 386 \end{bmatrix}$$
$$= \begin{bmatrix} Theater 1 \\ Theater 2 \end{bmatrix} \begin{bmatrix} C & A \\ 252 & 789 \\ 259 & 761 \end{bmatrix}$$



Adding & Subtracting Matrices

You can perform matrix addition on matrices with equal dimensions.

$$\mathbf{a.} \begin{bmatrix} 9 & 0 \\ -4 & 6 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad \mathbf{b.} \begin{bmatrix} 3 & -8 \\ -5 & 1 \end{bmatrix} + \begin{bmatrix} -3 & 8 \\ 5 & -1 \end{bmatrix} \\ = \begin{bmatrix} 9+0 & 0+0 \\ -4+0 & 6+0 \end{bmatrix} \qquad = \begin{bmatrix} 3+(-3) & -8+8 \\ -5+5 & 1+(-1) \end{bmatrix} \\ = \begin{bmatrix} 9 & 0 \\ -4 & 6 \end{bmatrix} \qquad = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$