KEY

Unit 2 Day 1
MATRICES

MATRIX OPERATIONS

Warm-Up
Basic Matrix Practice

1. $\begin{bmatrix} 4 & -a \\ 6 & -3 \end{bmatrix} + \begin{bmatrix} -6 & -5 \\ 7a & 3 \end{bmatrix}$

2. $5\begin{bmatrix} 1 & -2 & x \\ 4 & y & 1 \\ 0 & -5 & x^2 \end{bmatrix}$

3. $\begin{bmatrix} 1 & 3 & -2 \\ 4 & 0 & 5 \end{bmatrix} - \begin{bmatrix} 2 & -1 & 5b \\ 6c & 4 & -3 \end{bmatrix}$

4. $3\begin{bmatrix} -3 & 0 \\ 4 & 5 \end{bmatrix}$

Warm-Up ANSWERS! Matrix Addition & Subtraction:

1. $$\begin{bmatrix} 4 & -a \\ 6 & -3 \end{bmatrix} + \begin{bmatrix} -6 & -5 \\ 7a & 3 \end{bmatrix} = \begin{bmatrix} -2 & -a-5 \\ 7a+6 & 0 \end{bmatrix}$$

3. $$\begin{bmatrix} 1 & 3 & -2 \\ 4 & 0 & 5 \end{bmatrix} - \begin{bmatrix} 2 & -1 & 5b \\ 6c & 4 & -3 \end{bmatrix} = \begin{bmatrix} -1 & 4 & -2-5b \\ 4-6c & -4 & 8 \end{bmatrix}$$

Warm-Up ANSWERS! Scalar Multiplication:

2. $$5\begin{bmatrix} 1 & -2 & x \\ 4 & y & 1 \\ 0 & -5 & x^2 \end{bmatrix} = \begin{bmatrix} 5 & -10 & 5x \\ 20 & 5y & 5 \\ 0 & -25 & 5x^2 \end{bmatrix}$$

4. $$3\begin{bmatrix} -3 & 0 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} -9 & 0 \\ 12 & 15 \end{bmatrix}$$

Questions About HW?

Questions About Matrices
Basics HW?

Unit 2 Day 1
NOTES

BASIC MATRIX OPERATIONS
& APPLICATIONS

* I'll go through these 1st slides quickly as they are mostly from last night's HW
→ maybe highlight or add to front side of last night's HW & suggest to students

Unit 2 Day 1
NOTES

BASIC MATRIX OPERATIONS & APPLICATIONS

Remember...

Matrix: a rectangular array of numbers or variables used to organize data.

$$A = \begin{pmatrix} 2 & -4 \\ -3 & 5 \\ 1 & 6 \end{pmatrix}$$

Typically, we name matrices with capital letters!

The numbers in the matrix are called elements.
There are 6 elements in this matrix.

Remember...

Matrix dimensions tell how many ROWS & COLUMNS there are in the matrix.

Dimensions & Notation

- A $_{3 \times 2}$
- **REMEMBER:**
  RC cola
  Rows by columns

$$\begin{array}{r} \text{rows} \rightarrow \\ \rightarrow \\ \rightarrow \end{array} A = \begin{pmatrix} 2 & -4 \\ -3 & 5 \\ 1 & 6 \end{pmatrix}$$
$$\uparrow \; \uparrow \text{ columns}$$

Rows run horizontally & columns run vertically.

The dimensions of two matrices can determine whether or not they may be added, subtracted, or multiplied.

A matrix of m rows and n columns is called a matrix with **dimensions m x n**

You Try Examples: Find the dimensions.

1.) $\begin{bmatrix} 2 & -3 & 4 \\ -1 & \frac{1}{2} & \pi \end{bmatrix}$ **2 X 3**

2.) $\begin{bmatrix} -3 & 8 & 9 \\ \pi & -2 & 5 \\ -6 & 7 & 8 \end{bmatrix}$ **3 X 3**

3.) $\begin{bmatrix} 10 \\ -7 \end{bmatrix}$ **2 X 1**

4.) $\begin{bmatrix} -3 & 4 \end{bmatrix}$ **1 X 2**

Matrix Position

NEW

We can give the location of an element in a matrix by naming its row, then its column.

$$\begin{pmatrix} 2 & -4 & -8 & 4 \\ -3 & 5 & -6 & 3 \\ 1 & 6 & -2 & 0 \end{pmatrix}$$

3 is in position k_{24}

6 is in position k_{32}

4 is in position k_{14}

-2 is in position k_{33}

Why use matrices?

- Matrix algebra makes *mathematical expression and computation* easier.
- It allows you to get rid of cumbersome notation, concentrate on the concepts involved and understand where your results come from.
- Matrices are used to represent real-world data such as the *habits* or *traits of populations.*

Special Matrices

NEW

Some matrices have special names because of what they look like.

a) **Row matrix**: only has 1 row. $Ex)$ $\begin{bmatrix} -3 & 4 \end{bmatrix}$

b) **Column matrix**: only has 1 column. $Ex)$ $\begin{bmatrix} 10 \\ -7 \end{bmatrix}$

c) **Square matrix**: has the same number of rows and columns. $Ex)$ $\begin{bmatrix} 10 & -2 \\ -7 & 3 \end{bmatrix}$

d) **Zero matrix**: contains all zeros. $Ex)$ $\begin{bmatrix} 0 & 0 \end{bmatrix}$

Remember...
Matrix Addition & Subtraction

- You can add or subtract matrices if they have the same dimensions (same number of rows and columns).
- To do this, you add (or subtract) the corresponding numbers (numbers in the same positions).
- If a matrix operation is not possible for a problem, the solution is called undefined.

Ex:

$$\begin{bmatrix} 2 & -4 \\ 5 & 0 \\ 1 & -3 \end{bmatrix} - \begin{bmatrix} -1 & -2 & 3 \\ 0 & 1 & -3 \end{bmatrix} = \textbf{\textit{Undefined}}$$

NEW

Properties of Matrix Addition

- Matrix addition **IS** commutative

  A + B = B + A

  Order does NOT Matter!

- Matrix addition **IS** associative

  A + (B + C) = (A + B) + C

  Grouping does NOT Matter!

Remember...

Scalar Multiplication

- To do this, multiply each entry in the matrix by the number outside (called the scalar).

  This is like distributing a number to a polynomial.

Example:

$$4\begin{bmatrix} 2 & -4 \\ 5 & 0 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} 8 & -16 \\ 20 & 0 \\ 4 & -12 \end{bmatrix}$$

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MATRIX MULTIPLICATION

Matrix Multiplication

- Matrix Multiplication is **NOT Commutative**! Order matters!
- You can multiply matrices only if the number of columns in the first matrix equals the number of rows in the second matrix.

$$\text{2 columns} \quad \begin{bmatrix} 2 & 3 \\ -5 & 6 \\ 9 & -7 \end{bmatrix} \bullet \begin{bmatrix} 1 & -2 & 0 \\ 3 & 4 & -5 \end{bmatrix} \longleftarrow \text{2 rows}$$

3 x 2   2 x 3   = **3 x 3 resulting matrix**

if =, then cross off those values to see NEW size (size of answer matrix)

Matrix Multiplication

- Take the numbers in the first row of matrix #1. Multiply each number by its corresponding number in the first column of matrix #2. Total these products.

Do 1st • 1st + 2nd • 2nd +...

$$\begin{bmatrix} 2 & 3 \\ -5 & 6 \\ 9 & -7 \end{bmatrix} \bullet \begin{bmatrix} 1 & -2 & 0 \\ 3 & 4 & -5 \end{bmatrix}$$

$$2 \bullet 1 + 3 \bullet 3 = 11$$

The result, 11, goes in row 1, column 1 of the answer. Repeat with row 1, column 2; row 1 column 3; row 2, column 1; ...

Continued on the next slide....

Let's Try!

To fill in element k_{13}, we use row 1 of 1st matrix with column 3 of 2nd matrix

Since answer is in spot k_{11}, use 1st row of 1st matrix, and 1st column of 2nd matrix to get it

$$\begin{bmatrix} 2 & 3 \\ -5 & 6 \\ 9 & -7 \end{bmatrix} \bullet \begin{bmatrix} 1 & -2 & 0 \\ 3 & 4 & -5 \end{bmatrix}$$

$(3 \times \cancel{2}) \bullet (\cancel{2} \times 3) =$ matrix Answer is 3×3 size

$$\begin{bmatrix} \underset{k_{11}}{2(1)+3(3)} & \underset{k_{12}}{2(-2)+3(4)} & \underset{k_{13}}{2(0)+3(-5)} \\ \underset{k_{21}}{-5(1)+6(3)} & \underset{k_{22}}{-5(-2)+6(4)} & \underset{k_{23}}{-5(0)+6(-5)} \\ \underset{k_{31}}{9(1)+-7(3)} & \underset{k_{32}}{\underset{-18+-28}{9(-2)+-7(4)}} & \underset{k_{33}}{9(0)+-7(-5)} \end{bmatrix}$$

$$\begin{bmatrix} 11 & 8 & -15 \\ 13 & 34 & -30 \\ -12 & -46 & 35 \end{bmatrix}$$

Let's Try Answer!

$$\begin{bmatrix} 2 & 3 \\ -5 & 6 \\ 9 & -7 \end{bmatrix} \bullet \begin{bmatrix} 1 & -2 & 0 \\ 3 & 4 & -5 \end{bmatrix} = \begin{bmatrix} 11 & 8 & -15 \\ 13 & 34 & -30 \\ -12 & -46 & 35 \end{bmatrix}$$

You Try!

$$\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 3 & -9 & 2 \\ 5 & 7 & -6 \end{bmatrix}$$

$$\begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix}$$

You Try! Answer

$$\begin{bmatrix}2 & -1\\ 3 & 4\end{bmatrix} \times \begin{bmatrix}3 & -9 & 2\\ 5 & 7 & -6\end{bmatrix}$$

$2 \times \cancel{2}$ $\cancel{2} \times 3$

$$\begin{bmatrix}2(3) + -1(5) & 2(-9) + -1(7) & 2(2) + -1(-6)\\ 3(3) + 4(5) & 3(-9) + 4(7) & 3(2) + 4(-6)\end{bmatrix}$$

$$= \begin{bmatrix}1 & -25 & 10\\ 29 & 1 & -18\end{bmatrix}$$

2×3

Matrix Multiplication

B A
$(3 \times 2)(2 \times 1)$

Find AB and BA given $A = \begin{bmatrix}5\\ -2\end{bmatrix}$ and $B = \begin{bmatrix}2 & 1\\ -9 & 0\\ 10 & -5\end{bmatrix}$

AB is *undefined.* A 2 x 1 and 3 x 2 cannot be multiplied.

$$BA = \begin{bmatrix}2 & 1\\ -9 & 0\\ 10 & -5\end{bmatrix} \bullet \begin{bmatrix}5\\ -2\end{bmatrix} = \begin{bmatrix}8\\ -45\\ 60\end{bmatrix}$$

$(3 \times \cancel{2})(\cancel{2} \times 1)$ 3×1

$$\begin{bmatrix}\underbrace{2(5) + 1(-2)}_{} \; (10 + -2)\\ -9(5) + 0(-2) \;\; (-45 + 0)\\ 10(5) + -5(-2) \;\; (50 + 10)\end{bmatrix} = \begin{bmatrix}8\\ -45\\ 60\end{bmatrix}$$

Matrix Multiplication Properties

- Matrix multiplication is **NOT** commutative

  AB≠BA order DOES matter

- Matrix multiplication **IS** associative

  A(BC)=(AB)C grouping does NOT matter

- Matrix multiplication **IS** distributive

  A(B+C)=AB+AC

  (A+B)C=AC+BC

Matrix Applications Handout #1 (Back Side)

2nd Find GOAL of problem... we'll want that as labels for answer matrix (NOT "inner dimensions" that go away!!)

1st Find repeated part... we'll want that as columns on 1st & rows on 2nd... so it cancels out

Two softball teams submit equipment lists for the season.
Women's Team: 12 bats, 45 balls, 15 uniforms
Men's Team: 15 bats, 38 balls 17 uniforms

Each bat costs $21, each ball costs $4, and each uniform costs $30.

Use matrix multiplication to find the total cost of equipment for each team.

$$\begin{array}{c|ccc} & \text{Bat} & \text{Ball} & \text{Unif} \\ \hline W & 12 & 45 & 15 \\ M & 15 & 38 & 17 \end{array} \cdot \begin{array}{c|c} & \text{cost} \\ \hline \text{Bat} & 21 \\ \text{Ball} & 4 \\ \text{Unif} & 30 \end{array} = \begin{array}{c|c} W & 12(21)+45(4)+15(30) \\ M & 15(21)+38(4)+17(30) \end{array}$$

$$\underbrace{(2\times \cancel{3})}_{\text{women, men}} \cdot \underbrace{(\cancel{3}\times 1)}_{\text{cost}} = \underbrace{2\times 1}_{\text{women \& men team costs}}$$

bat, ball, uniforms

$$= \begin{array}{c|c} & \text{Cost} \\ \hline W & 882 \\ M & 977 \end{array}$$

womens team costs $882 and men's team costs $ 977

Application Answer

$$\begin{array}{c} \text{EQUIPMENT} \\ \begin{array}{r} \\ \text{Women's team} \\ \text{Men's team} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Bats} & \text{Balls} & \text{Uniforms} \end{array} \\ \begin{bmatrix} 12 & 45 & 15 \\ 15 & 38 & 17 \end{bmatrix} \end{array} \end{array} \qquad \begin{array}{c} \text{COST} \\ \begin{array}{r} \\ \text{Bats} \\ \text{Balls} \\ \text{Uniforms} \end{array} \begin{array}{c} \text{Dollars} \\ \begin{bmatrix} 21 \\ 4 \\ 30 \end{bmatrix} \end{array} \end{array}$$

The total cost of equipment for each team can now be obtained by multiplying the equipment matrix by the cost per item matrix. The equipment matrix is 2×3 and the cost per item matrix is 3×1, so their product is a 2×1 matrix.

$$\begin{bmatrix} 12 & 45 & 15 \\ 15 & 38 & 17 \end{bmatrix} \begin{bmatrix} 21 \\ 4 \\ 30 \end{bmatrix} = \begin{bmatrix} 12(21) + 45(4) + 15(30) \\ 15(21) + 38(4) + 17(30) \end{bmatrix} = \begin{bmatrix} 882 \\ 977 \end{bmatrix}$$

The labels for the product matrix are as follows.

$$\begin{array}{c} \text{TOTAL COST} \\ \begin{array}{r} \\ \text{Women's team} \\ \text{Men's team} \end{array} \begin{array}{c} \text{Dollars} \\ \begin{bmatrix} 882 \\ 977 \end{bmatrix} \end{array} \end{array}$$

▶ The total cost of equipment for the women's team is \$882, and the total cost of equipment for the men's team is \$977.

Practice

Matrix Applications Handout #2 and 3

(*Back Side*)