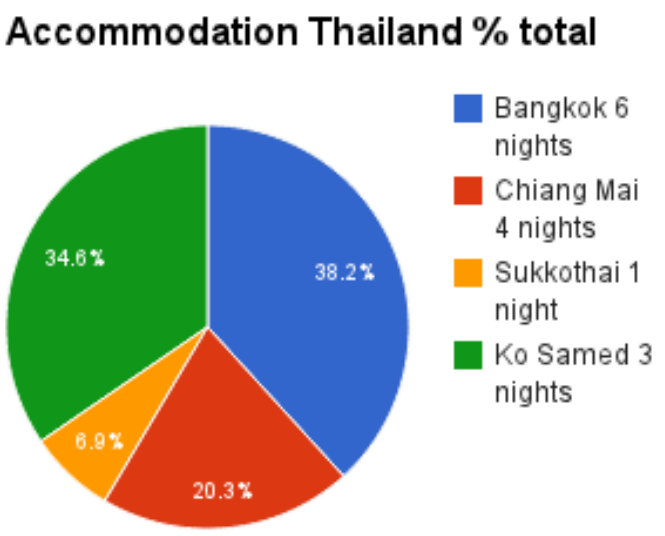
Day 1 Financial Math

INTEREST AND ANNUITIES



The next two days:

- 1) Recall interest and annuity formulas and how to use them.
- 2) Fill out a monthly budget for our friend Steph based on her salary, habits, possessions, etc. (= Tonight's HW)
- 3) Fill out a *personal* monthly budget based on typical living expenses, using an assigned salary and Real Life Scenario

Interest Compounded Annually

Suppose P is the principal amount of money invested in an account bearing an interest rate r at the end of each year. Let A_n represent the total amount in the account at the end of n years.

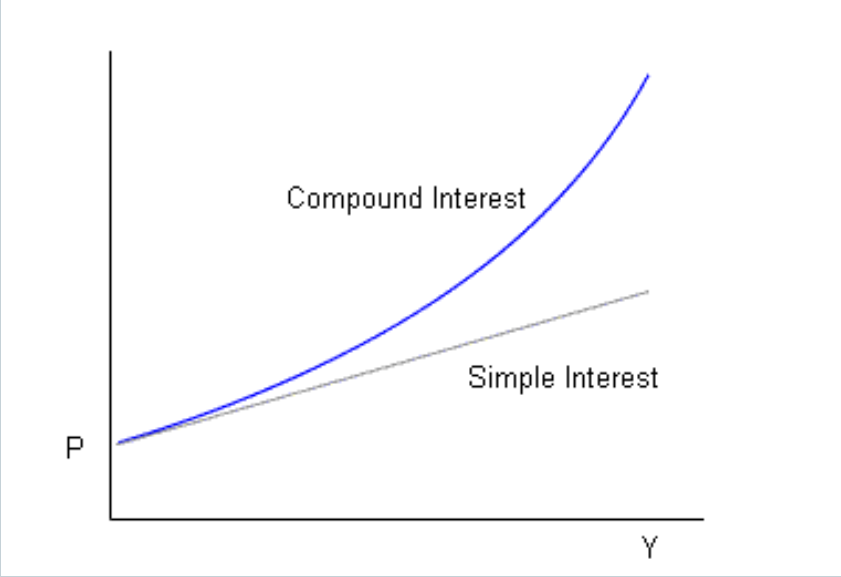
Time (years)	Amount in account
0	$A_0 = P$
1	$A_1 = P + Pr = P(1 + r)$
2	$A_2 = A_1 + A_1r = A_1(1 + r) = P(1 + r)^2$
3	$A_3 = A_2 + A_2r = A_2(1 + r) = P(1 + r)^3$
.	.
.	.
n	$\mathbf{A_n = P(1 + r)^n}$

Interest Compound k times a year

- Now, r/k is the interest rate, t is the number of years, and kt is the number of compounding periods.
- Thus, $A = P\left(1 + \frac{r}{k}\right)^{kt}$
- Notice that this is just a slight variation from the first formula $\mathbf{A_n = P(1 + r)^n}$, in which k = 1.

Compound Interest Formulas

- Interest Compounds *k times a year*: $A = P\left(1+\frac{r}{k}\right)^{kt}$
- Interest Compounds *Continuously*: $A = Pe^{rt}$



Annuities- FUTURE value

- So far, we've only considered the situation in which an investor has made a single deposit. Now, consider an investor making regular deposits monthly, quarterly, etc. This is an annuity.
- Sequence of equal, periodic payments.
- Net amount of money returned from an annuity is called the FUTURE VALUE.
- http://www.econedlink.org/interactives/index.php?iid=2

Future Value of an Annuity

- The future value, FV, of an annuity consisting of n equal periodic payments of R dollars at an ***interest rate i per compounding period (payment interval)*** is

$$FV = R\frac{(1+i)^n - 1}{i}$$

- This formula can be proven using the Annual Percentage Yield (APY) which we did not discuss.
- Use when **SAVING**.

$$i = \frac{r}{k} \qquad n = kt$$

Loans and Mortgages- **PRESENT** value

- Net amount of money put into an annuity.

- How does the bank determine what the periodic payments should be? It considers what would happen to the present value as an investment with interest compounding over the term of the loan and compares the result to the future value of the loan repayment annuity.

Present Value of an Annuity

- The present value PV of an annuity consisting of n equal payments of R dollars earning an ***interest rate i per period*** (payment interval) is

$$PV = R\frac{1-(1+i)^{-n}}{i}$$

- Use when **BUYING**.

$$i = \frac{r}{k} \qquad n = kt$$

Finance Formulas

$$i = \frac{r}{k} \qquad n = kt$$

Interest compounded k times per year	Interest compounded continuously
$A = P\left(1 + \frac{r}{k}\right)^{kt}$	$A = Pe^{rt}$
Future Value of an Annuity	**Present Value of an Annuity**
$FV = R\frac{(1+i)^n - 1}{i}$	$PV = R\frac{1-(1+i)^{-n}}{i}$

Which to use?!?!

Ask yourself…

Spending Money?

OR

Saving Money?

$$PV = R\frac{1-(1+i)^{-n}}{i}$$

Multiple deposits

$$FV = R\frac{(1+i)^{n}-1}{i}$$

Single deposit

"Compounded Continuously"

$$A = Pe^{rt}$$

Not continuously

$$A = P\left(1+\frac{r}{k}\right)^{kt}$$

EXAMPLES!

1) Suppose Lucy Cash invests $1000 at 6% interest compounded **annually**. What is the value of Lucy's investment after 8 years?

EXAMPLES ANSWERS!

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$$A = P(1+r)^t$$

$$A = 1000(1+.06)^8 = \$1593.85$$

EXAMPLES!

2) Richie Rich has $700 to invest at 2% annual interest compounded monthly. How long will it take for his investment to grow to $4000?

How much interest did he earn?

EXAMPLES ANSWERS!

2) Richie Rich has $700 to invest at 2% annual interest compounded monthly. How long will it take for his investment to grow to $4000?

$$A = P(1+\frac{r}{t})^{kt} \qquad 4000 = 700(1+\frac{.02}{12})^{12t} = 87.22\ years$$

How much interest did he earn?

4000-700 = $ 3,300

EXAMPLES!

3) Suppose Anita Lone invests $200 in her savings account at 0.9% annual interest compounded **continuously**. Find the value of her investment at the end of 6 years.

EXAMPLES ANSWERS!

3) Suppose Anita Lone invests $200 in her savings account at 0.9% annual interest compounded **continuously**. Find the value of her investment at the end of 6 years.

$A = Pe^{rt}$ $A = 200e^{(.009)(6)} = \$211.10$

EXAMPLES!

4) a. Buck is saving money for his spring break trip and the banks are offering great rates on their saving accounts. At the end of each quarter (report card time), Buck makes deposits of $200 into an account that pays 3% interest compounded quarterly. How much will Buck have in his account at the end of the year?

$$FV = R\frac{(1+i)^n - 1}{i}$$

Must divide interest by 4!

b. How much money did he contribute? How much interest did he earn?

c. Another bank is offering a rate of 4.5% interest compounded quarterly. How much will Buck have after 1 year?

EXAMPLES ANSWERS!

4) a. Buck is saving money for his spring break trip and the banks are offering great rates on their saving accounts. At the end of each quarter (report card time), Buck makes deposits of $200 into an account that pays 3% interest compounded quarterly. How much will Buck have in his account at the end of the year?

$$FV = R\frac{(1+i)^n - 1}{i} = 200\frac{\left(1+\frac{.03}{4}\right)^{4(1)} - 1}{\frac{.03}{4}} = \$809.05$$

Must divide interest by 4!

b. How much money did he contribute? How much interest did he earn?

200 x 4 = $ 800 $ 9.05

c. Another bank is offering a rate of 4.5% interest compounded quarterly. How much will Buck have after 1 year?

$ 813.60

More Examples

5) a. Mercedes purchases a car for $18,500. What are the monthly payments for a 4-year loan with a $2000 down payment if the annual interest (APR) is 2.9%?

$$PV = R\frac{1-(1+i)^{-n}}{i}$$

Must divide interest by 12!

b. What if the interest rate was 5.9%? How much more are you paying in interest?

More Examples ANSWERS

5) a. Mercedes purchases a car for $18,500. What are the monthly payments for a 4-year loan with a $2000 down payment if the annual interest (APR) is 2.9%?

$$PV = R\frac{1-(1+i)^{-n}}{i} \qquad 16500 = R\frac{1-\left(1+\frac{.029}{12}\right)^{-12(4)}}{\frac{.029}{12}} = \$364.49$$

Must divide interest by 12!

b. What if the interest rate was 5.9%? How much more are you paying in interest?

$ 364.49

386.75 – 364.49 = $ 22.26 more in interest

$$PV = R\frac{1-(1+i)^{-n}}{i}$$

More Examples

6) a. Haley Homes wants to buy a townhouse. She has nothing to put towards her down payment and the townhouse she likes is $140,000. If Corporate Mortgage Financial is offering her a loan with a 4.6% interest rate compounded monthly for 30 years, what is her monthly payment?

Must divide interest by 12!

b. After 30 years, how much does she actually end up paying for the townhouse?

c. How much does she pay in interest?

$$PV = R\frac{1-(1+i)^{-n}}{i}$$

More Examples ANSWERS

6) a. Haley Homes wants to buy a townhouse. She has nothing to put towards her down payment and the townhouse she likes is $140,000. If Corporate Mortgage Financial is offering her a loan with a 4.6% interest rate compounded monthly for 30 years, what is her monthly payment?

Must divide interest by 12!

$$140000 = R\frac{1-\left(1+\frac{.046}{12}\right)^{-12(30)}}{\frac{.046}{12}} = \$717.70$$

b. After 30 years, how much does she actually end up paying for the townhouse?

717.70 x 12 x 30 = $ 258,372.76

c. How much does she pay in interest?

258,372.76 - 140,000 = $ 118,372

If Time Allows, Today…

- We will start looking at Steph's budget using these formulas and common accounting sense
  - Day 2 Part A Handout

Tomorrow…

- We will check over Steph's budget using these formulas and common accounting sense
- Then you will create your own budget and expenses, using an assigned salary and Real Life Scenario.

Homework:

- Fill out a monthly budget for our friend Steph based on her salary, habits, possessions, etc.
- Refer to your notes as you work ☺
- This example will help GREATLY when you complete your own personal budget tomorrow.