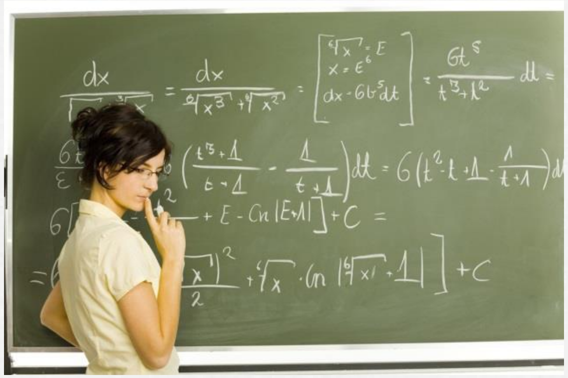
Day 1

Unit 5 – Intro to Derivatives

&

Limit Definition of Derivative

For Polynomials



Warm Up

1) Given $f(x) = x^2 - 2x + 1$, evaluate

a) $f(x+7)$ b) $f(x+h)$ c) $f(x+h) - f(x)$

2) Find $f(g(x))$ and its domain.

$$f(x) = \frac{x^2 - 4x}{x^2} \qquad g(x) = \sqrt{7 - x}$$

3) Find the slope given $h(2) = 3$ and $h(5) = -4$

4) Write an equation for the line in #3 in slope-intercept form.

Warm Up ANSWERS

1) Given $f(x) = x^2 - 2x + 1$, evaluate

a) $f(x+7)$ b) $f(x+h)$ c) $f(x+h) - f(x)$

$x^2 + 12x + 36$

$2hx + h^2 - 2h$

$x^2 + 2hx + h^2 - 2x - 2h + 1$

2) Find $f(g(x))$ and its domain.

$$f(x) = \frac{x^2 - 4x}{x^2} \qquad g(x) = \sqrt{7-x}$$

$$\frac{7 - x - 4\sqrt{7-x}}{7-x}$$

$$Domain: \ (-\infty, 7)$$

3) Find the slope given $h(2) = 3$ and $h(5) = -4$

$m = -7/3$

4) Write an equation for the line in #3 in slope-intercept form.

$$y - 3 = -\frac{7}{3}(x-2) \qquad y = -\frac{7}{3}x + \frac{23}{3}$$

Discuss HW

- PreRequisite Review for Unit 5 Handout

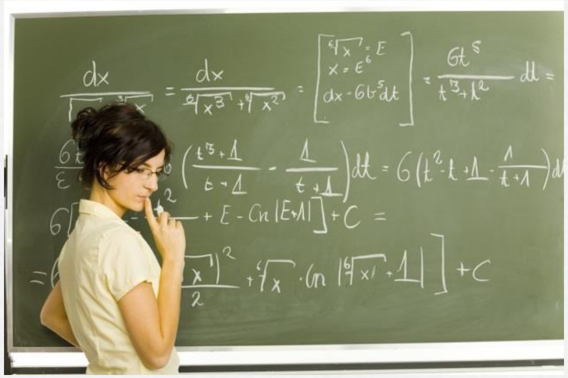
Tonight's HW

- Finish Classwork if necessary
- Packet p. 1

Notes Day 1

Intro to Derivatives

And Limit Defn. of Deriv.

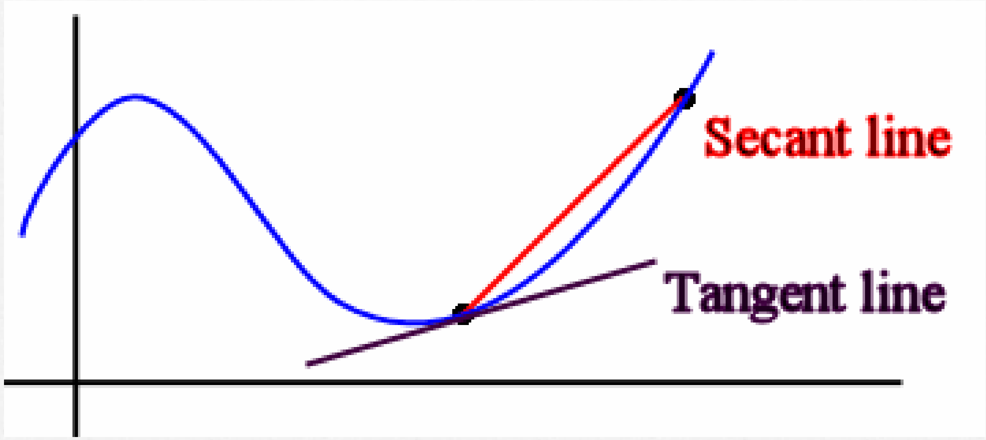


Introduction to Derivatives Webquest

- Answer the questions on the handout.
- Take Notes on other key points! ☺
- Be prepared to discuss afterwards.

Secant vs Tangent

- Tangent lines touch a curve at one point
- Slope at that one point is *instantaneous* rate of change.
- Secant lines cut through a curve at two points
- The slope of a secant line between those two points it is called the *average* rate of change.

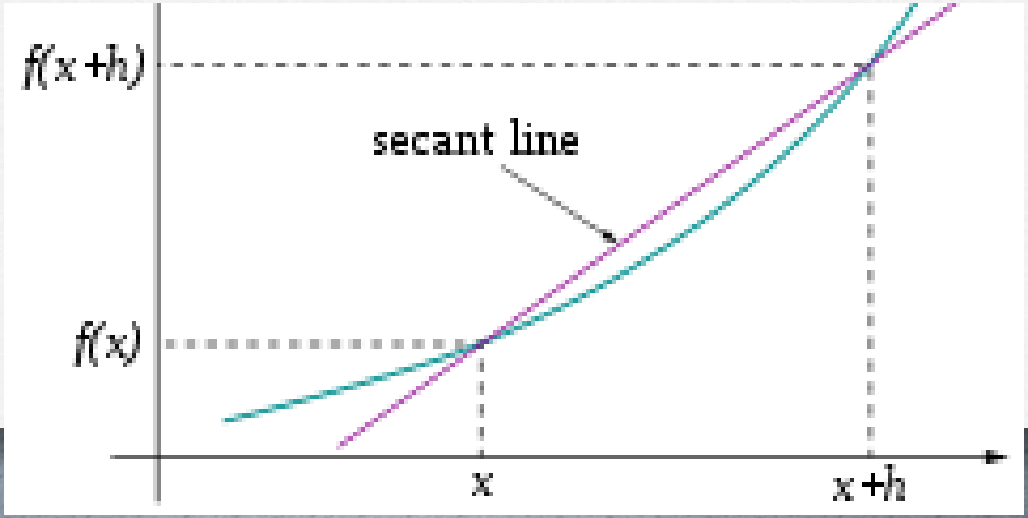


Slope – all the same!

$$m = rate\ of\ change = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{change in y}}{\text{change in x}} = \frac{\Delta y}{\Delta x}$$

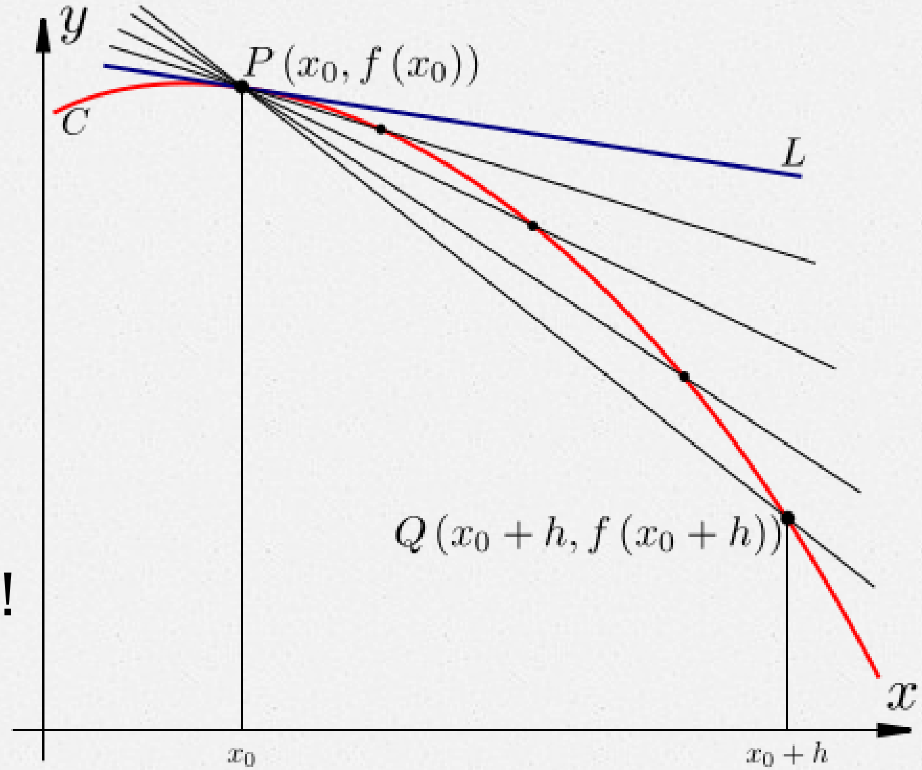
$$m = \frac{f(x_2) - f(x_1)}{x_2 - x_1} \quad where:\left(x_1, f(x_1)\right);\left(x_2, f(x_2)\right)$$

$$m = \frac{f(x+h) - f(x)}{x+h-x} \quad where:\left(x, f(x)\right);\left(x+h, f(x+h)\right)$$



Use Secant or Tangent?

- Tangent lines are REALLY hard to draw. So you can draw a secant line and calculate its slope as one point on the line gets closer and closer to the point of tangency (thus, making the secant line into the tangent line).
- Of course, the change in x (the h here) would be 0 if the two points actually made it on top of each other.
- That's where the idea of limits comes in here!



Limit Definition of Derivative!

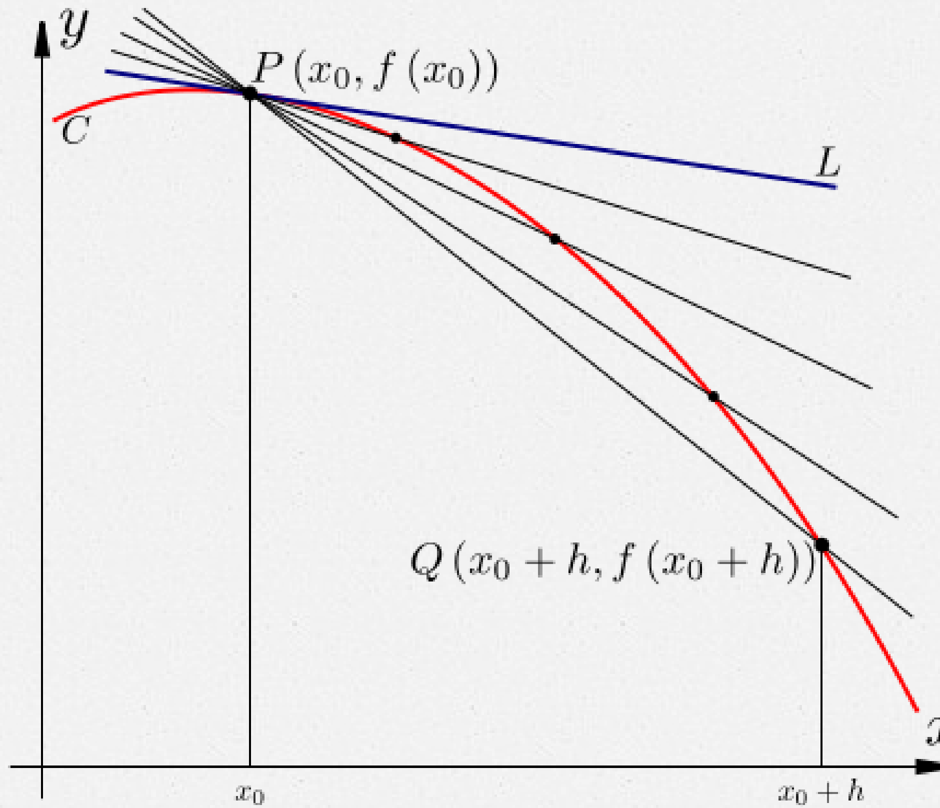
Building off of the last slope you wrote down

$$m = \frac{f(x+h) - f(x)}{x+h-x}$$

The Limit Definition of Derivative is

$$f'(x) = \frac{d}{dx} f(x) =$$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$



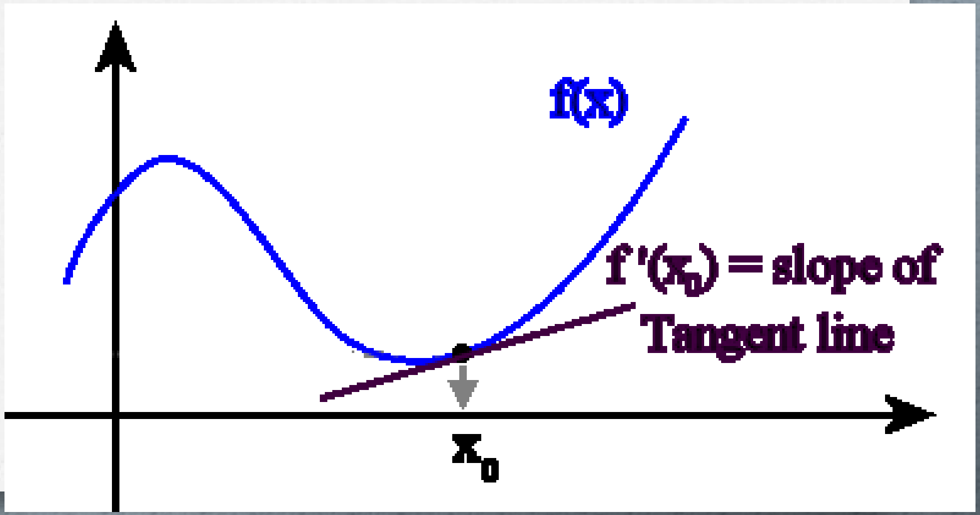
Limit definition of derivative

The derivative with respect to x is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

The derivative is

- The slope of the tangent line at a single point on the curve.
- The Instantaneous rate of change



What do we need to write an equation of a line?

1) One point and slope; then use point slope formula $y - y_1 = m(x - x_1)$

2) Two points; then compute the slope and use one of the points in the point slope formula

→ We'll do this at a later date!!

Notation of Derivative

- Given $f(x) = 3x^2 + 5x$.

$$\frac{dy}{dx}, \quad y', \quad f'(x)$$

We could say…

- $f'(x) = 6x + 5$ or
- $y' = 6x + 5$ or
- $\frac{dy}{dx} = 6x + 5$

f' or y' is called
"f prime or
y prime"

$\frac{dy}{dx}$ is said
"derivative of y
with respect to x"

Why are there 2 notations for derivatives?

- Due to history!
- There were 2 founders of Calculus – at the same time – Leibniz and Newton.
- https://youtu.be/axZTv5YJssA

Example 1:

Evaluate the derivative using the limit definition of derivatives. $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

- Function: $f(x) = 4x + 9$

$= 4$

Example 2:

Evaluate the derivative using the limit definition of derivatives.

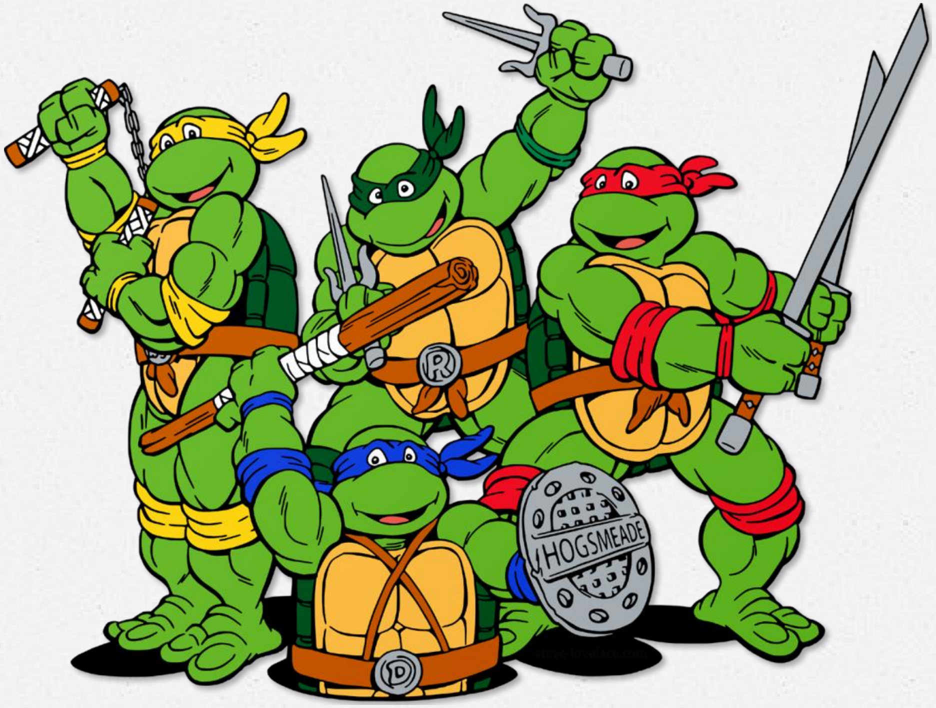
$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- Function: $f(x) = x^2 + 2x + 3$

$$= 2x + 2$$

Classwork: Packet p.2

- What did the ninja turtles say when handed the expression….?



HW:
Packet p. 1

On next slides…

Introduction to Limits videos

College Math Lecture Videos

- Be attentive!
- Take Notes!
- Be prepared to discuss at the end!

College Math Lecture Videos

- https://www.youtube.com/watch?v=jbIQW0gkgxo
- MIT- Lecture 1 Single Variable Calculus

- https://www.youtube.com/watch?v=54_XRjHhZzI
- NC State- Introduction to Limits

Next slides…

Saved for Day 2 discussion and practice for Spring ’18 and Fall ’18 and Spring ‘19

Example 3:

Evaluate the derivative using the limit definition of derivatives.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- Function: $f(x) = \sqrt{x-2}$

For Square Root problems, you must use the conjugate!

$$= \frac{1}{2\sqrt{x-2}}$$

Example 4:

Evaluate the derivative using the limit definition of derivatives.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- Function: $f(x) = \dfrac{1}{x+1}$

$$= \frac{-1}{x^2 + 2x + 1}$$