Review
Day 10

Unit 2 - Matrices & Game Theory

Warm-Up

Phones OFF & in the pockets!!

A certain kind of bug lives only four weeks. 70% of the bugs survive from the first week of life to the second. 65% of those who make it to the second week also survive into the third. 50 % survive to the fourth week. No bugs live beyond four weeks. On average, 12 newborn bugs are produced by these bugs who make it to the fourth week. The population of bugs that you receive in the lab has 16 one week old, 10 two week old, 9 three week old, and 4 four week old bugs. (Hint: Create a table from the info.)

1. Construct a Leslie Matrix for the life cycle of this bug.

2. What is the initial distribution matrix for this scenario?

3. What is the new population distribution after 8 cycles?

4. What is the total bug population after 14 cycles?

Warm-Up ANSWERS

A certain kind of bug lives only four weeks. 70% of the bugs survive from the first week of life to the second. 65% of those who make it to the second week also survive into the third. 50 % survive to the fourth week. No bugs live beyond four weeks. On average, 12 newborn bugs are produced by these bugs who make it to the fourth week. The population of bugs that you receive in the lab has 16 one week old, 10 two week old, 9 three week old, and 4 four week old bugs. (Hint: Create a table from the info.)

1. **Construct a Leslie Matrix for the life cycle of this bug.**

Weeks	Birth Rate	Survival Rate
1	0	.70
2	0	.65
3	0	.50
4	12	0

$$L = \begin{bmatrix} 0 & 0.70 & 0 & 0 \\ 0 & 0 & 0.65 & 0 \\ 0 & 0 & 0 & 0.50 \\ 12 & 0 & 0 & 0 \end{bmatrix}$$

Warm-Up ANSWERS

The population of bugs that you receive in the lab has 16 one week old, 10 two week old, 9 three week old, and 4 four week old bugs.

2. **What is the initial distribution matrix for this scenario?**

$$P_0 = \begin{bmatrix} 16 & 10 & 9 & 4 \end{bmatrix}$$

3. **What is the new population distribution after 8 cycles?**

$$P_8 = P_0 \bullet L^8 = \begin{bmatrix} 119.25 & 74.53 & 67.08 & 29.81 \end{bmatrix}$$

4. **What is the total bug population after 14 cycles?**

1996.59 bugs

$$L = \begin{bmatrix} 0 & 0.70 & 0 & 0 \\ 0 & 0 & 0.65 & 0 \\ 0 & 0 & 0 & 0.50 \\ 12 & 0 & 0 & 0 \end{bmatrix}$$

Homework Questions from last night?! (Packet p. 11 and 12)

Tonight's Homework

- Test Review Handout
- Complete AT LEAST Stations B & E
  And 1 other station

- Complete Quiz 2 Corrections
- follow proper format and do ALL the parts

Classwork = Review Stations & Corrections!

1st) Start Quiz Corrections

- Staple Correction paper on TOP of Quiz
- Use proper format (see side board)
- Ask questions

2nd) Start Review Stations

- You will turn in this tomorrow STAPLED to BACK of Test Review sheet
- You MUST complete AT LEAST Stations B & E & another of your choice – so you may want to do B and E 1st ☺
- Show your work on NEW Notebook paper

Unit 2 Matrix Test Topics

1. Strictly Determined Games: Find **maximin**, **minimax** and **saddle points -** be able to interpret each based on the context.
2. Non-Strictly Determined Games: Find **payoff matrix**, **best strategy** for row and column player, and **expected value** for row player.
3. Markov Chains: Create **transition matrix** and **initial-state matrix** and interpret values after a certain number of cycles.
4. Leslie Matrix: Create Leslie matrix to find **population distribution, total population, long term growth rate,** and time when a **maximum population** or **future minimum population** is reached.
5. Matrix Operations: Perform calculations and interpret properly, especially **Matrix multiplication** and **Scalar multiplication.**
6. Matrix Applications: Be able to use matrix operations to solve **word problem applications** and **systems of equations**.

Tonight's Homework

- Test Review Handout
- Complete AT LEAST Stations B & E
  And 1 other station

- Complete Quiz 2 Corrections
- follow proper format and do ALL the parts

Did not use next slide for Fall ‘18

- Had done a Markov problem day before for warm-up

Warm-Up

There are 20 students in student council and every week they bring snacks to their meeting. This week 8 brought chips, 7 brought drinks and 5 brought dessert. 18% of those who brought chips to the first meeting brought chips again and 42% brought drinks. Of those that brought drinks, 35% brought drinks again and the rest brought dessert to the next meeting. And of those that brought dessert to the first meeting, 26% brought dessert again and 48% brought chips.

a. What is the initial matrix for the student council?

b. What is the transition matrix for the student council?

c. Approximately how many students will bring drinks to the 4^{th} meeting??

d. In the long run, how many of these students will bring each item to a meeting?

Warm-Up Answers

There are 20 students in student council and every week they bring snacks to their meeting. This week 8 brought chips, 7 brought drinks and 5 brought dessert. 18% of those who brought chips to the first meeting brought chips again and 42% brought drinks. Of those that brought drinks, 35% brought drinks again and the rest brought dessert to the next meeting. And of those that brought dessert to the first meeting, 26% brought dessert again and 49% brought drinks.

a. What is the initial matrix for the student council?

$$D_0 = \begin{bmatrix} 8 & 7 & 5 \end{bmatrix}$$

b. What is the transition matrix for the student council?

$$T = \begin{array}{c} \\ Drinks \\ Chips \\ Dessert \end{array} \begin{array}{c} \begin{array}{ccc} Drinks & Chips & Dessert \end{array} \\ \begin{bmatrix} 0.35 & 0 & 0.65 \\ 0.42 & 0.18 & 0.40 \\ 0.49 & 0.25 & 0.26 \end{bmatrix} \end{array}$$

c. Approximately how many students will bring desserts to the 4th meeting?

8.86 so approximately 8-9 students (Find D_4, then its dessert column.)

d. In the long run, how many of these students will bring each item to a meeting?

Around 8.43 (8-9) will bring drinks, 2.70 (2-3) will bring chips, and 8.86 (8-9) will bring desserts

Practice

Suppose that Sol and Tina change their game. Now, Sol will win 3 cents if both players show Heads, Sol will win 1 cent if both players show Tails, and Sol will pay 2 cents if one shows Heads and the other shows Tails.

a. Write a payoff matrix for this scenario.

b. Use the row matrix $\begin{bmatrix} p & 1-p \end{bmatrix}$ to find Sol's best strategy for this game.

c. Use the column matrix $\begin{bmatrix} q \\ 1-q \end{bmatrix}$ to find Tina's best strategy for this game.

d. Set up a tree diagram to compute the probabilities of each of the four outcomes for this game.

e. Prepare a probability distribution chart for Sol's winnings.

f. Find Sol's expectation for this game.

Practice Answers

a. $\begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix}$

$$\begin{bmatrix} \frac{3}{8} & \frac{5}{8} \end{bmatrix} \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} \frac{3}{8} \\ \frac{5}{8} \end{bmatrix} = \begin{bmatrix} -\frac{1}{8} \end{bmatrix} \approx [-.125]$$

e. Sol should play heads 3 of the 8 times and tails 5 of the 8 times.
Tina should play heads 3 of the 8 times and tails 5 of the 8 times.

f. Sol is expected to lose 1 penny for every 8 games he plays.