ICM Unit 1 Day 10

Conditional Probability

Warm Up Day 10:

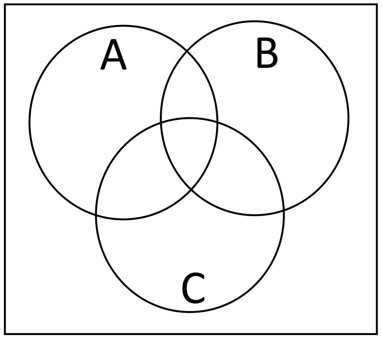
Fill in the table below about eye color:

Eye Color	Black	Brown	Blue	Green	Gray	Total
Female	20	30	10	15	10	
Male	25	15	12	20	10	
Total						

Turn in Cards and Dice Lab to the Silver Tray!

1. Find
   a. $P(Brown \cup Green)^c$
   b. $(Black \cup Blue) \cap Male$
   c. $Blue \cup Green \cup Female$

2. Shade a Venn Diagram for $(A \cup B)^c \cap (C \cap A)^c$



3. If 3 coins are chosen in succession from a bag consisting of 12 dimes and 7 quarters, what is the probability of choosing exactly 2 dimes? (Hint: order matters)

Given $f(x) = x^2 - 7$, evaluate

4. $f(x - 3) + 2$

5. $f(3x^4) + 2$

Warm Up ANSWERS

Fill in the table below about eye color:

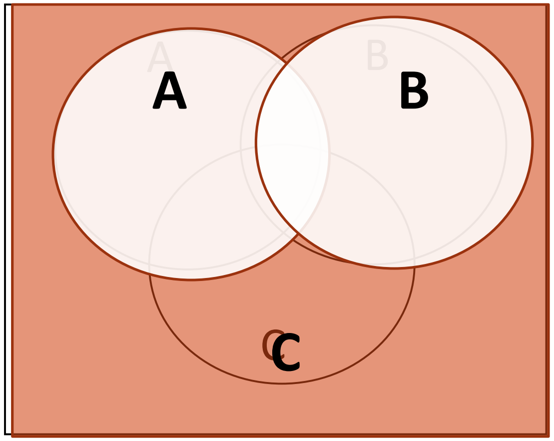
Eye Color	Black	Brown	Blue	Green	Gray	Total
Female	20	30	10	15	10	**85**
Male	25	15	12	20	10	**82**
Total	**45**	**45**	**22**	**35**	**20**	**167**

1. Find a. $P(Brown \cup Green)^c$ $\frac{87}{167} = 0.521$

b. $(Black \cup Blue) \cap Male$ 37

c. $Blue \cup Green \cup Female$ 117

2. Shade a Venn Diagram for
$$(A \cup B)^c \cap (C \cap A)^c$$



Warm Up ANSWERS

3. If 3 coins are chosen in succession from a bag consisting of 12 dimes and 7 quarters, what is the probability of choosing exactly 2 dimes? (Hint: order matters)

$$P(DDQ) + P(DQD) + P(QDD)$$

$$\frac{12}{19}\cdot\frac{11}{18}\cdot\frac{7}{17} + \frac{12}{19}\cdot\frac{7}{18}\cdot\frac{11}{17} + \frac{7}{19}\cdot\frac{12}{18}\cdot\frac{11}{17} = 0.4768$$

Given $f(x) = x^2 - 7$, evaluate

4. $f(x - 3) + 2$

$(x - 3)^2 - 7 + 2$

$(x - 3)(x - 3) - 7 + 2$

$x^2 - 6x + 9 - 5$

$\mathbf{x^2 - 6x + 4}$

5. $f(3x^4) + 2$

$(3x^4)^2 - 7 + 2$

$\mathbf{9x^8 - 5}$

Riddle of the Day

I'm there once in a minute, twice in a moment but never in a thousand years. Who am I?

Answer: The letter M!

Homework Questions?

Announcements

*Probability Cards & Dice Lab is due today! Turn in to the Silver Tray if you haven't yet!

*Test is Monday – start your Review

Let's look at #25 on HW p.13

Five black balls and four white balls are placed in an urn. Two balls are then drawn in succession. What is the probability that the second ball drawn is a white ball if…

a. The second ball is drawn without replacement?

$$\{BW\},\{WW\} = \frac{5}{9}\bullet\frac{4}{8} + \frac{4}{9}\bullet\frac{3}{8} = \frac{32}{72} = 0.444$$

b. The first ball is replaced before the second is drawn?

$$\{BW\},\{WW\} = \frac{5}{9}\bullet\frac{4}{9} + \frac{4}{9}\bullet\frac{4}{9} = \frac{36}{81} = 0.444$$

Tonight's Homework

Packet p. 14-15

& Start Review for Test

Conditional Probability

Section 7.5 Part

Ex: Two cards are drawn without replacement from a well-shuffled deck of 52 cards.

these events are ***dependent***

- What is the probability that the 1st card drawn is an ace?

$$\frac{4}{52} = \frac{1}{13}$$

- What is the probability that the 2nd card drawn is an ace **given** that the 1st card was not an ace?

this "given" statement tells us a fact that we ***know*** AND must use!

$$\frac{4}{51}$$

4 aces in deck

51 cards total

- What is the probability that the 2nd card drawn is an ace **given** that the 1st card drawn is an ace?

$$\frac{3}{51}$$

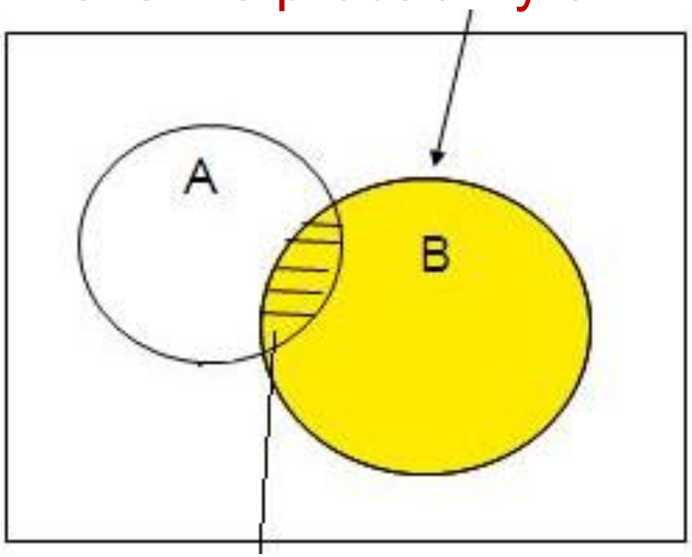
3 aces in deck

51 cards total

- **<u>Conditional Probability</u>**- when the probability of an event ***<u>is affected</u>*** by the knowledge of other information relevant to the event.

- **<u>Notation</u>:** **P(A | B)** → the probability of event A ***given*** that event B has occurred

<u>Given</u> B ***<u>has</u>*** happened, what's the probability of A?



$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\text{Number of elements in A and B}}{\text{Number of elements in B}}$$

In a conditional probability problem, the sample space is "reduced" to the "space" of the given outcome (e.g. if given B, we now just care about the probability of A occurring "inside" of B)

Ex: You roll a fair die. Find the probability that you roll a 2 given that your roll is an even number.

$$P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{\text{Number of elements in A and B}}{\text{Number of elements in B}}$$

$$P(2|Even)=\frac{P(2\,and\,even)}{P(even)}=\frac{1/6}{3/6}=\frac{1}{3}$$

Ex: A pair of fair dice is cast. What is the probability that the sum of the numbers you roll is 7 if it is known that one of the numbers is a 5?

Watch out! Given problems can be in disguise! Here it doesn't say given, but tells us a known fact. So it's still a "given" problem!:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\text{\# of elements in A and B}}{\text{\# of elements in B}}$$

Die	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

$$P(sum\ of\ 7 \mid 5)$$

$$= \frac{P(sum\ of\ 7\ and\ a\ 5)}{P(a\ 5)}$$

$$= \frac{2/36}{11/36} = \frac{2}{11}$$

- **Ex:** In a test conducted by the US Army, it was found that of 1000 new recruits (600 men and 400 women) 50 of the men and 4 of the women were red-green color blind. Given that a recruit selected at random from this group is red-green color blind, what is the probability that the recruit is male?

$$P(M \mid C) = \frac{P(M \cap C)}{P(C)} = \frac{P(\text{male and colorblind})}{P(\text{colorblind})}$$

$$= \frac{50/1000}{54/1000} = \frac{0.05}{0.054} = 0.9259$$

Ex: In Mr. Jonas' homeroom, 70% of the students have brown hair, 25% have brown eyes, and 5% have both brown hair and brown eyes. A student is excused early to go to a doctor's appointment. If the student has brown hair, what is the probability that the student also has brown eyes?

$$P(A|B) = \frac{P(\text{A and B})}{P(B)}$$

P(brown eyes | brown hair)

= P(brown eyes and brown hair)/P(brown hair)

= 0.05/0.7

= 0.071

The probability of a student having brown eyes given he or she has brown hair is 7.1%

You Try!

- 1) In North Carolina 62% of all adults own a car and 43% of all adults own a car and a house. What is the probability that an adult owns a house given that they own a car?

- 2) In Mrs. Walden's class, 65% of the students have brown hair, 30% have green eyes, and 8% have both brown hair and green eyes. A student is excused early to go to a doctor's appointment. If the student has brown hair, what is the probability that the student also has green eyes?

- 3) Let E and F be events in a sample space such that P(E)=0.7, P(F) = 0.2 and $P(E \cap F) = 0.15$

Find: $a.\ P(E|F)$ $b.\ P(F|E)$

You Try Answers!

- 1) In North Carolina 62% of all adults own a car and 43% of all adults own a car and a house. What is the probability that an adult owns a house given that they own a car?

P(own a car and house | own a car)

= 0.43/0.62

= 0.694

69.4%

You Try Answers!

2) In Mrs. Walden’s class, 65% of the students have brown hair, 30% have green eyes, and 8% have both brown hair and green eyes. A student is excused early to go to a doctor's appointment. If the student has brown hair, what is the probability that the student also has green eyes?

$$P(A|B) = \frac{P(\text{A and B})}{P(B)}$$

P(green eyes | brown hair)

= P(green eyes and brown hair)/P(brown hair)

= 0.08/0.65

= 0.123

The probability of a student having green eyes given he or she has brown hair is 12.3%

You Try Answers:

- 3) Let E and F be events in a sample space such that P(E)=0.7, P(F) = 0.2 and $P(E \cap F) = 0.15$

  Find:

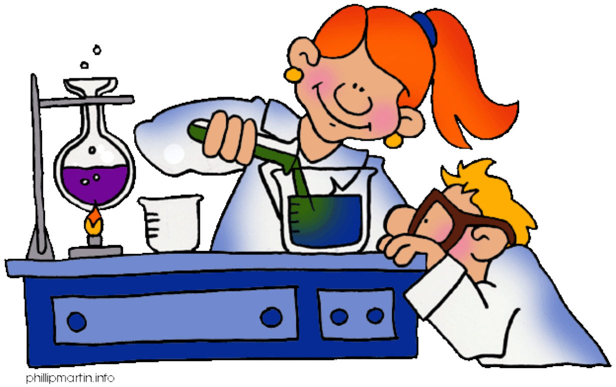
  $a.\ P(E|F)$

  $b.\ P(F|E)$

$$\frac{P(E \cap F)}{P(F)} = \frac{.15}{.2} = .75$$

$$\frac{P(E \cap F)}{P(E)} = \frac{.15}{.7} = .214$$

A **COMPOUND EVENT** is an event that is the result of more than one outcome.



Example: What is the probability that you forgot to do your homework AND there will be a pop quiz in class?

Example: If you flip a coin and roll a die, what is the probability of getting tails and an even number?

To calculate probability of compound events:

1st) MAKE A TREE DIAGRAM.
place probabilities on the branches
and words after the branches.

2nd) MULTIPLY ALONG THE BRANCHES ON THE TREE. (Like the counting principle, you multiply to find the overall probability)

To fill in the probabilities for the branches not stated in our problem, we'll need to use the "**complement**" of each probability. $P(A^c) = 1 - P(A)$

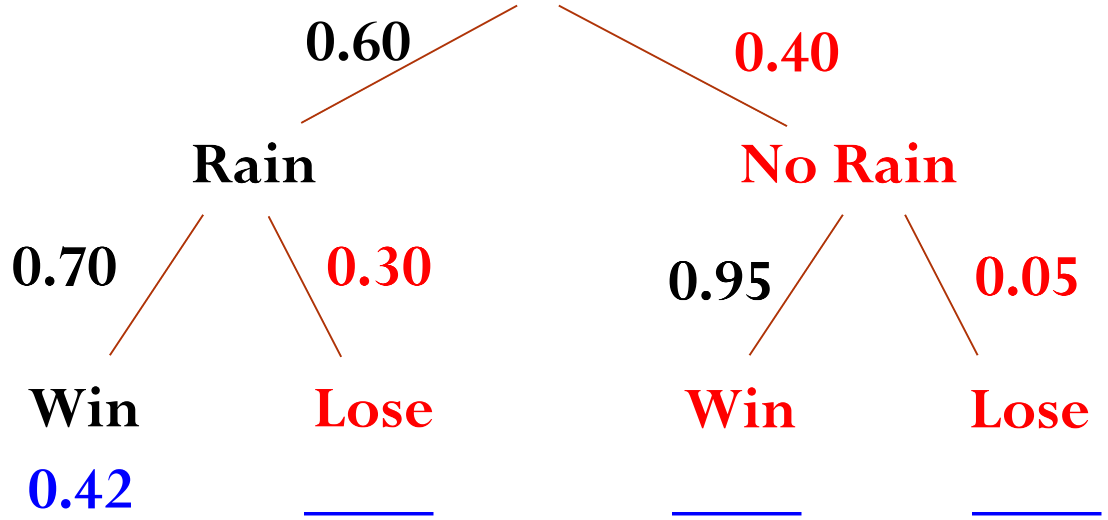
To calculate probability of compound events:

1st) MAKE A TREE DIAGRAM. place probabilities on the branches and words after the branches.

2nd) MULTIPLY ALONG THE BRANCHES ON THE TREE. (Like the counting principle, you multiply to find the overall probability)

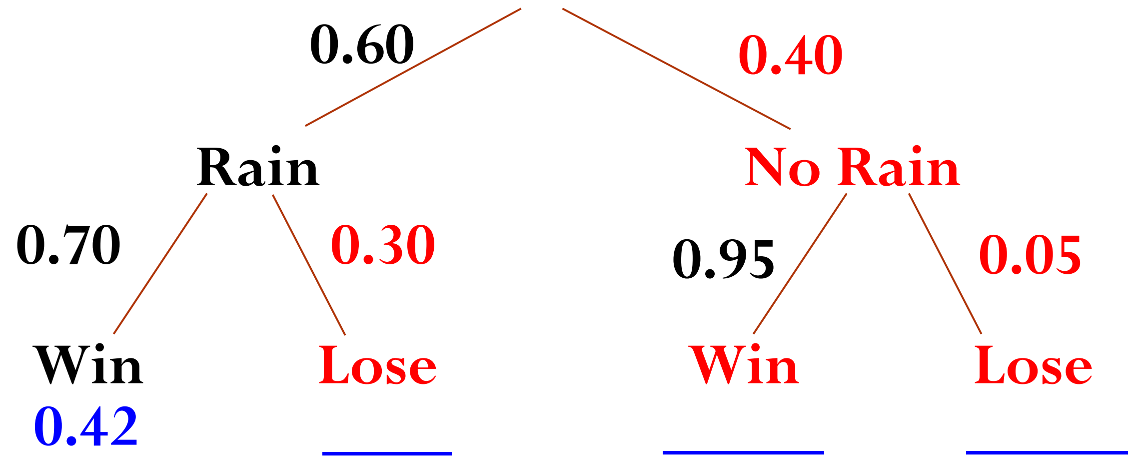
Example: Create a Tree Diagram for the following scenario:

There is a 60% chance of rain on Wednesday.
If it rains, the track team has a 70% chance of winning.
If it doesn't rain, there is a 95% chance that the track team will win.



We'll fill these ____ in on the next slide!

Ex 3) Use the tree diagram to fill in the remaining outcomes and probabilities:

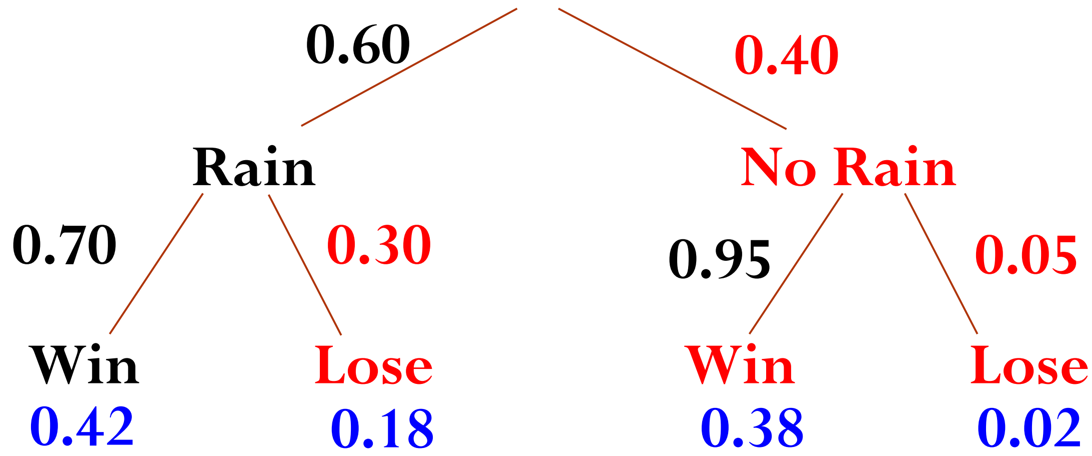


Outcome	Calculations	Probability
Rain and Track Team Wins	(.60)(.70) = 0.42	42%

Ex 4 a) What is the probability that the track team wins?

b) What is the probability the team wins given that it rains = P(win | rain)?

Ex 3) Use the tree diagram to fill in the remaining outcomes and probabilities:



Outcome	Calculations	Probability
Rain and Track Team Wins	(.60)(.70) = 0.42	42%
Rain and Track Team Loses	(.60)(.30) = 0.18	18%
No Rain and Track Team Wins	(.40)(.95) = 0.38	38%
No Rain and Track Team Loses	(.40)(0.05) = 0.02	2%

Ex 4 a) What is the probability that the track team wins?

42% + 38% = 80 %

b) What is the probability the team wins given that it rains

= P(win | rain)? **0.42/0.6 =70 %**

A Test for Independent Events

- *If **A** & **B** are independent events, then

$$P(A\,|\,B) = P(A) \text{ and } P(B\,|\,A) = P(B)$$

- **BE CAREFUL NOT TO CONFUSE INDEPENDENT EVENTS WITH MUTUALLY EXCLUSIVE EVENTS.**

- **From Ex 3:** Are winning and rain independent events?

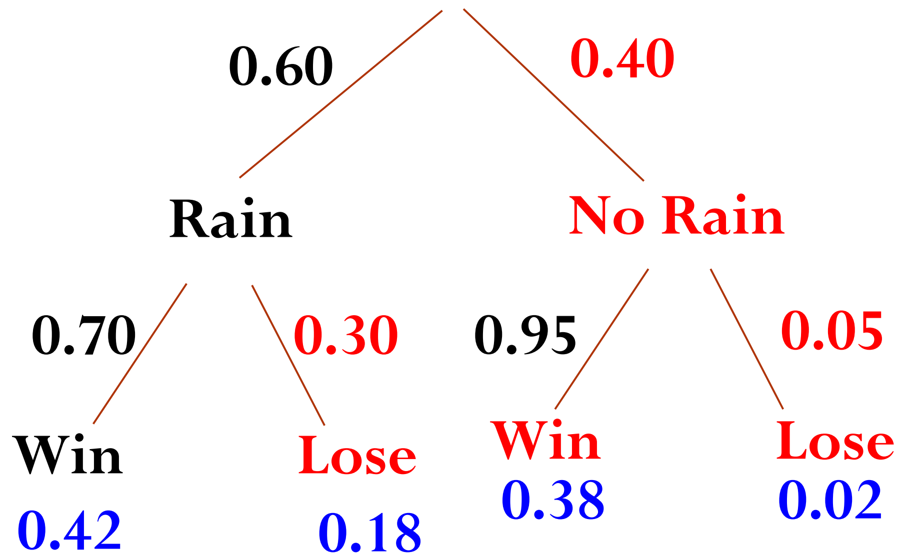
P(win | rain)

$$= \frac{P(win\ and\ rain)}{P(rain)} = \frac{(.7)(.6)}{(.6)} = .7$$

$BUT\ \ P(win) = 0.8$

So, $P(win\,|\,rain) \neq P(win)$

therefore, winning and rain are dependent events.



You Try! Create a Tree Diagram for the following scenario:

There is a 70% chance of thunderstorms on Thursday.
If it storms, the swimming pool will close 92% of the time.
If it doesn't storm, the pool will be open 97% of the time.

a) Create a tree diagram for the scenario listing all possibilities and probabilities.

b) Find the probability that it storms and the pool is closed.

c) Find the probability that the pool is open.

d) If the pool is open, find the probability of storms.

e) Are the storms and pool being open independent events? Explain.

You Try! Create a Tree Diagram for the following scenario:

There is a 70% chance of thunderstorms on Thursday.
If it storms, the swimming pool will close 92% of the time.
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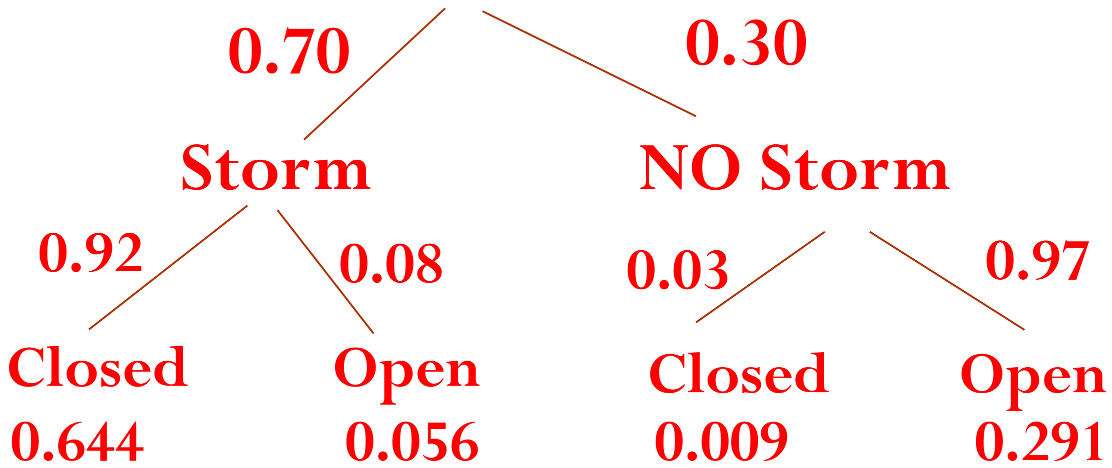


b) Find the probability that it storms and the pool is closed.

$$(.7)(.92) = 64.4\ \%$$

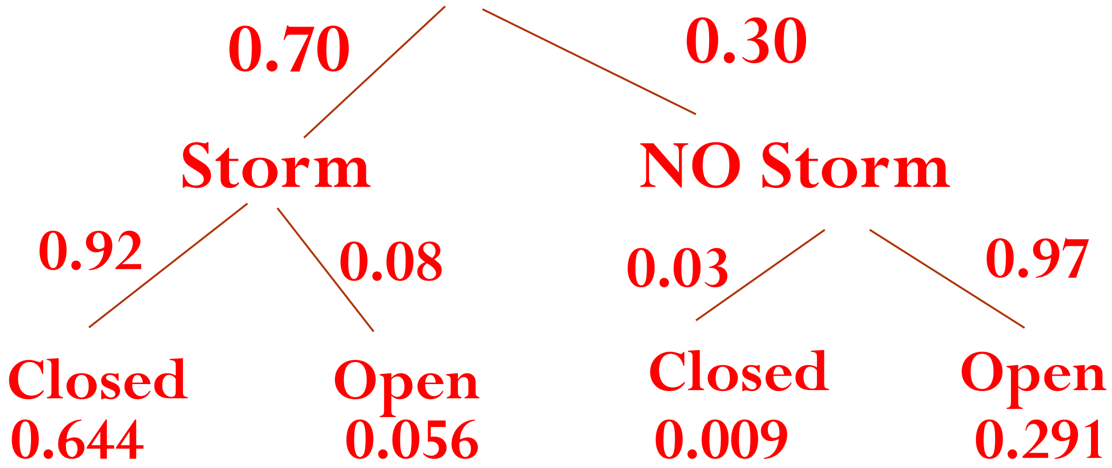
c) Find the probability that the pool is open.

$$(.7)(.08) + (.3)(.97) = 34.7\ \%$$

d) Find P(storms | pool is open).

$$\frac{(.7)(.08)}{(.7)(.08)+(.3)(.97)} = .161 = 16.1\%$$

You Try ANSWERS! There is a 70% chance of thunderstorms on Thursday. If it storms, the swimming pool will close 92% of the time. If it doesn't storm, the pool will be open 97% of the time.



d) Find P(storms | pool is open).

$$\frac{(.7)(.08)}{(.7)(.08)+(.3)(.97)} = .161 = 16.1\%$$

e) Are the storms and pool being open independent events? Explain.

$$P(Storms\,|\,open) = 0.161 = 16.1\%$$

$$BUT \;\; P(storms) = 0.7 \qquad So, \;\; P(storms\,|\,open) \neq P(storms)$$

therefore, storms and the pool being open are *<u>de</u>pendent events.*

You Try #2! Suppose you manage a restaurant that serves pizza that is either stuffed crust or original crust, and it can have meat or be vegetarian. From your experience you know that of stuffed crust pizza bought, 75% of them have meat, and of the original crust bought, 70% are vegetarian. Only 4 out of 10 customers buy stuffed crust pizza. Express your answers as percents rounded to the tenths place.

a) Create a tree diagram for the scenario displaying all the possibilities and probabilities.

b) P(stuffed and vegetarian)

c) P(vegetarian | stuffed)

d) P(vegetarian)

e) P(stuffed |vegetarian)

f) P(pizza with meat)

g) P(stuffed | meat)

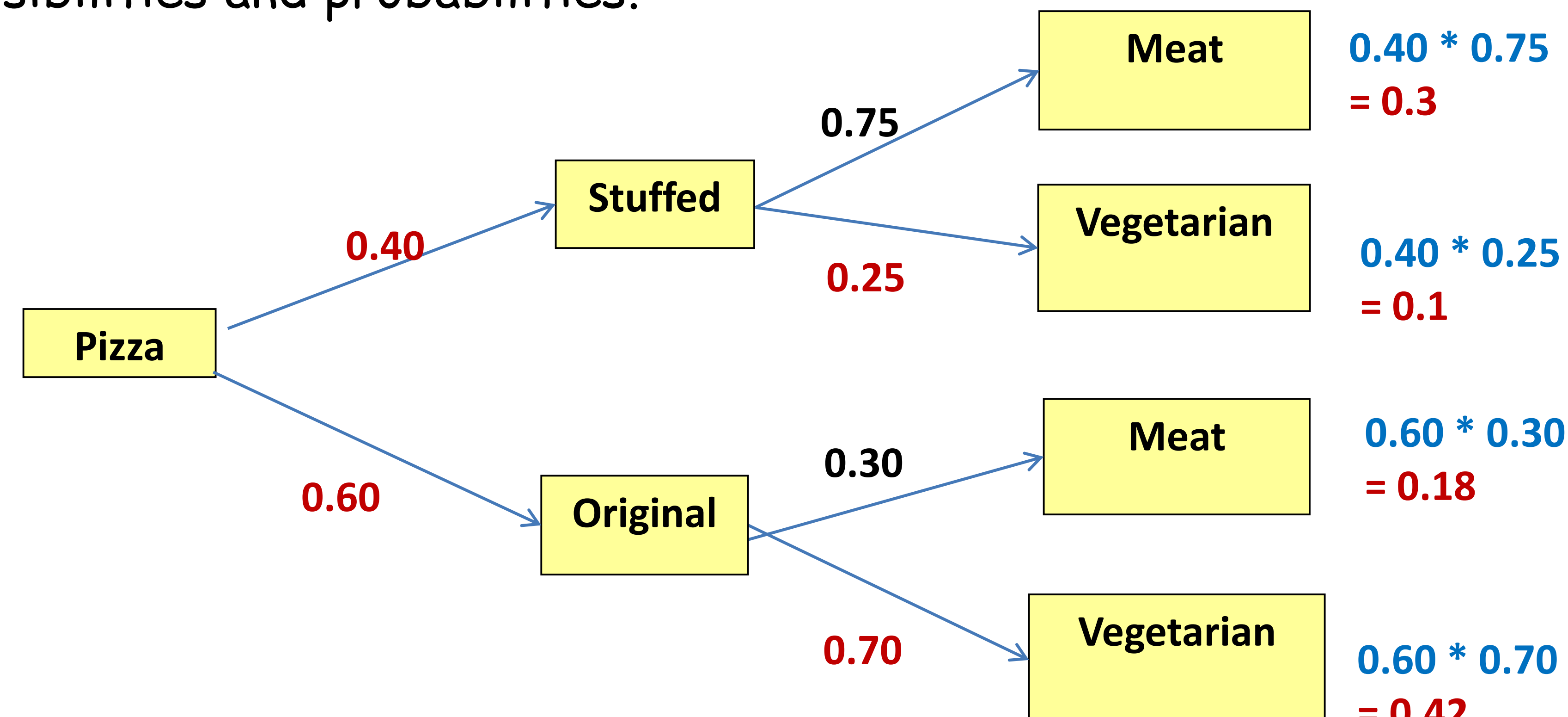
h) If a person orders original crust, what is the probability they choose it with meat?

i) Of the original crust, what is the probability someone orders vegetarian?

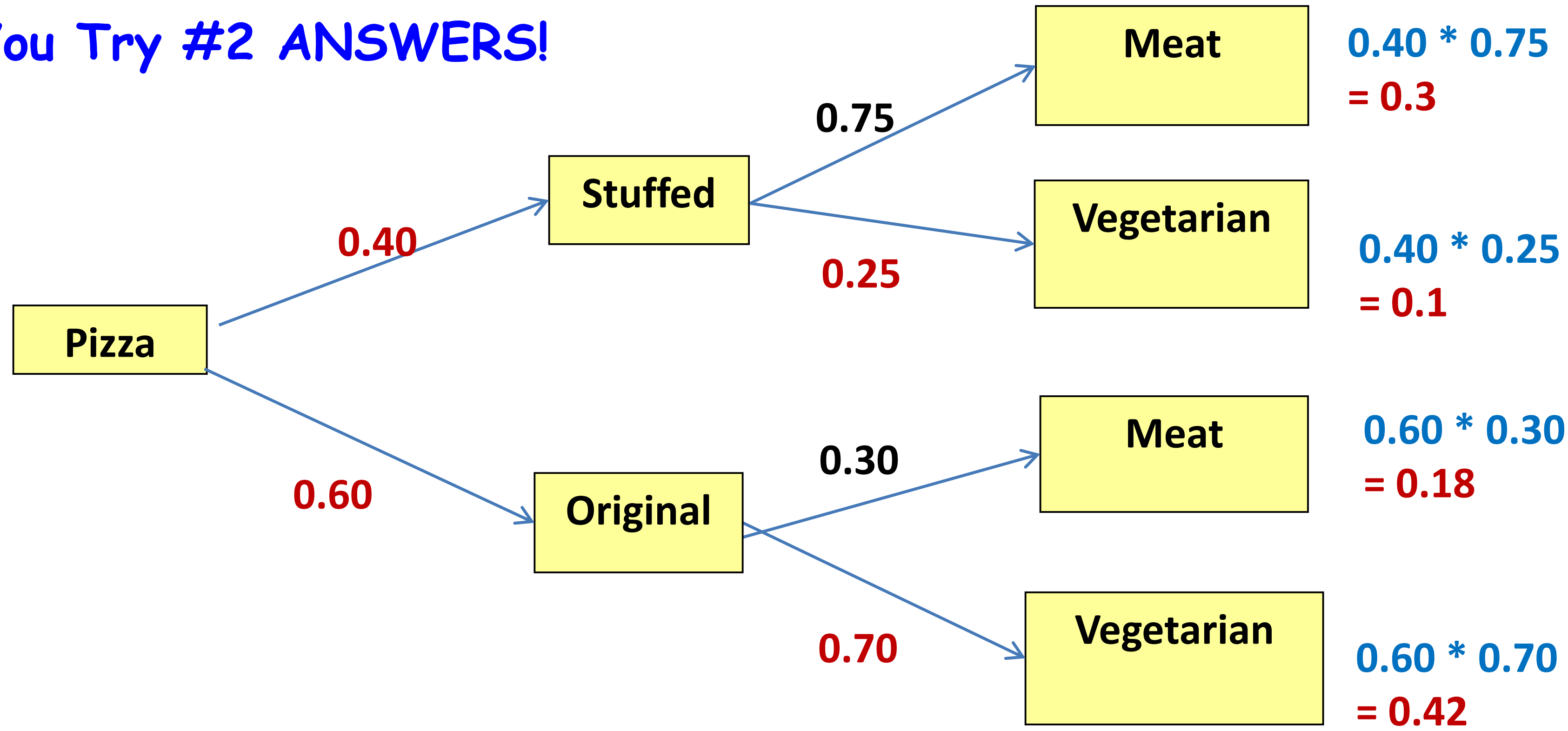
Hint: None of these probabilities should be the same!

You Try #2 ANSWERS! Suppose you manage a restaurant that serves pizza that is either stuffed crust or original crust, and it can have meat or be vegetarian. From your experience you know that of stuffed crust pizza bought, 75% of them have meat, and of the original crust bought, 70% are vegetarian. Only 4 out of 10 customers buy stuffed crust pizza.

a) Create a tree diagram for the scenario displaying all the possibilities and probabilities.



You Try #2 ANSWERS!



b) P(stuffed and vegetarian)

$$(.4)(.25) = .1$$

$$10\%$$

c) P(vegetarian | stuffed)

$$\frac{(.4)(.25)}{(.4)} = .25 = 25\%$$

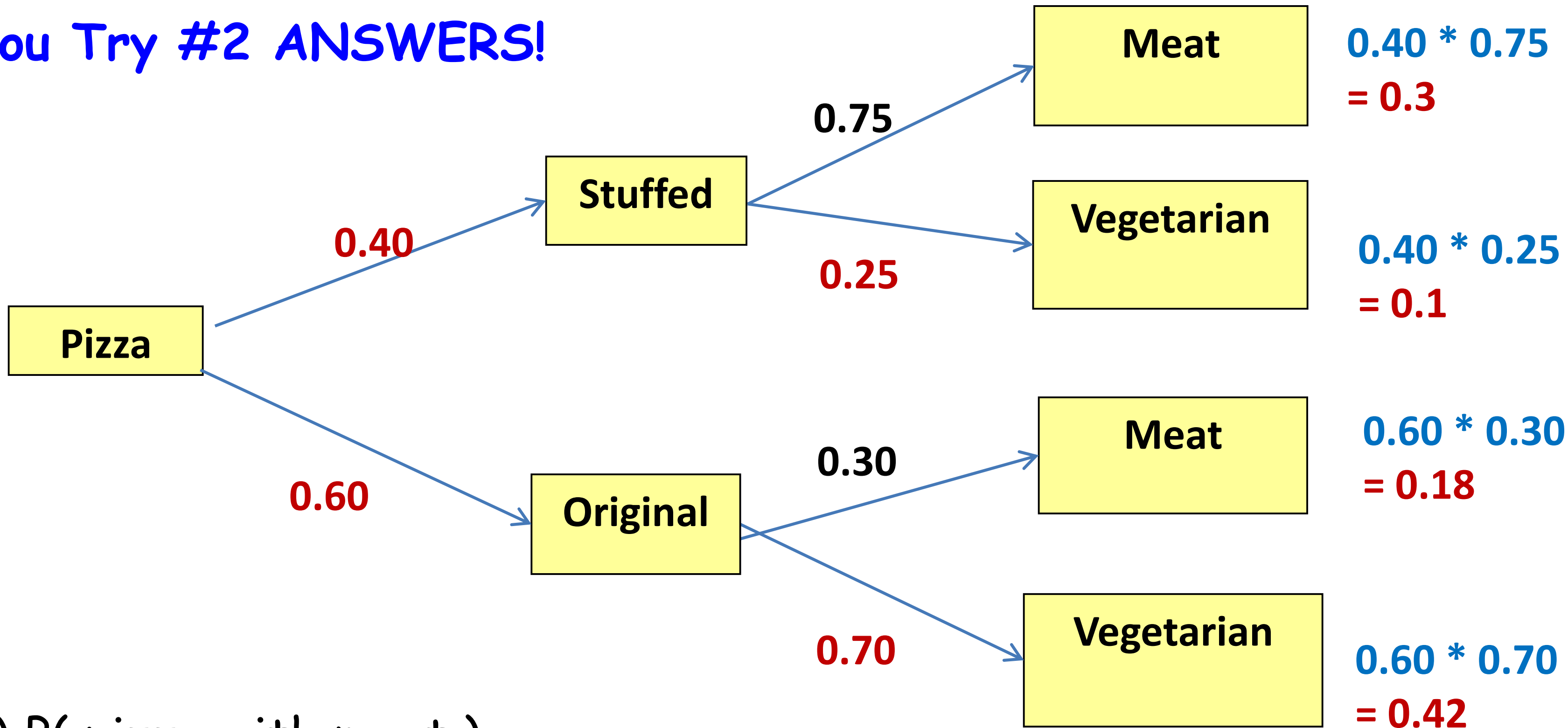
d) P(vegetarian)

$$(.4)(.25) + (.6)(.7) = .52 = 52\%$$

e) P(stuffed |vegetarian)

$$\frac{(.4)(.25)}{(.4)(.25) + (.6)(.7)} = .192 \quad = 19.2\%$$

You Try #2 ANSWERS!



f) P(pizza with meat)

$$(.4)(.75)+(.6)(.3)=.48$$

48%

g) P(stuffed | meat)

$$\frac{(.4)(.75)}{(.4)(.75)+(.6)(.3)}=.625=62.5\%$$

h) If a person orders original crust, what is the probability they choose it with meat?

30%

i) Of the original crust, what is the probability someone orders vegetarian?

70%