Unit 1 Day 1

Set Operations & Venn Diagrams

Honors ICM

- Get out your signed syllabus form
- Get out paper and a pencil for notes!

- Has everyone accessed the website?

Math Riddles

- Mr. Smith has 4 daughters. Each of his daughters has a brother.

  How many children does Mr. Smith have?

He has 5 children, all of the daughters have the same 1 brother.

Math Riddles

- John has been hired to paint the numbers 1 through 100 on 100 apartments.

  How many times with he have to paint 8?

  20 times
  (8, 18, 28, 38, 48, 58, 68, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 98)

Notes Day 1

- Continued….

- **<u>Set builder notation:</u>** A rule that describes the definite property (properties) an object x must satisfy to be part of the set.

- EX. $A = \{x \mid x \text{ is an even integer}\}$

  "x such that x is an even integer"

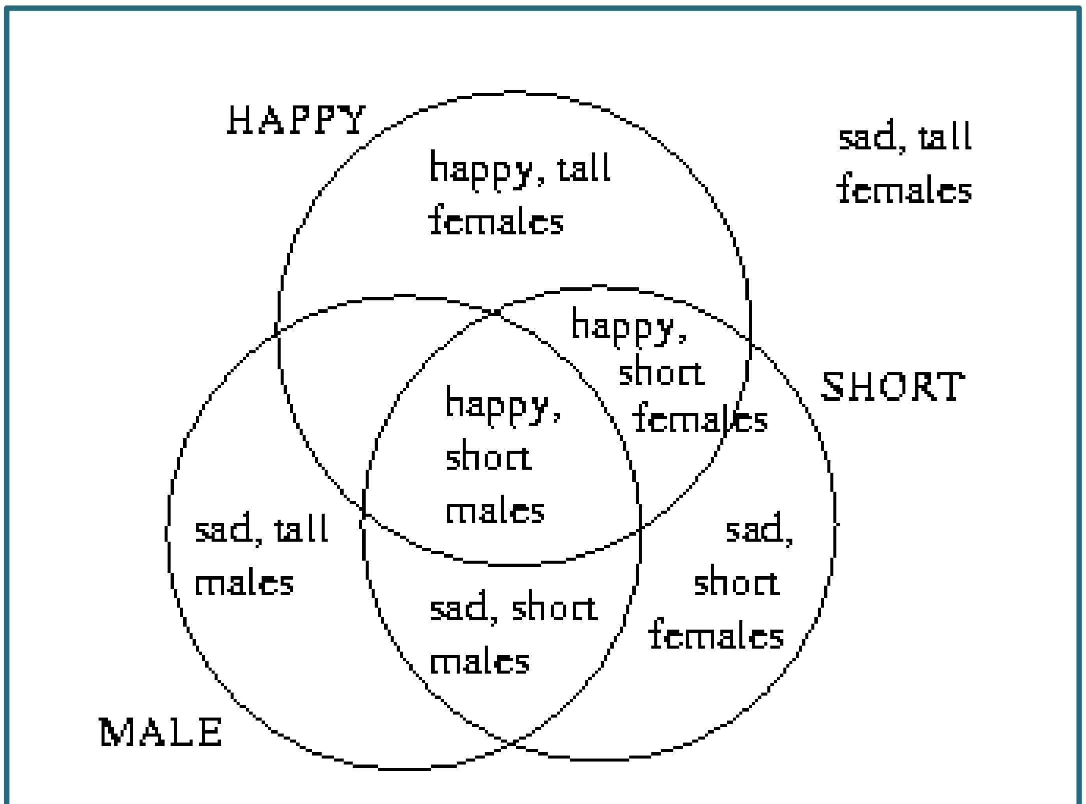
- EX. The set B of all letters of the alphabet

  Roster notation: $B = \{a, b, c, \ldots, x, y, z\}$

  Set-builder notation: $B = \{x \mid x \text{ is a letter of the English alphabet}\}$

Definitions continued

- <u>VENN DIAGRAMS</u>: Diagrams that show all possible logical relations between a finite collection of sets



Set Operations: The ways in which sets can be combined to yield other sets.

- UNION: The union of two sets is the set obtained by combining the members of each.
- INTERSECTION: The intersection of two sets is the set of elements common to A and B.
- COMPLEMENT: a **complement** of a set *A* refers to elements not in A.

More about these terms on the next slides ->

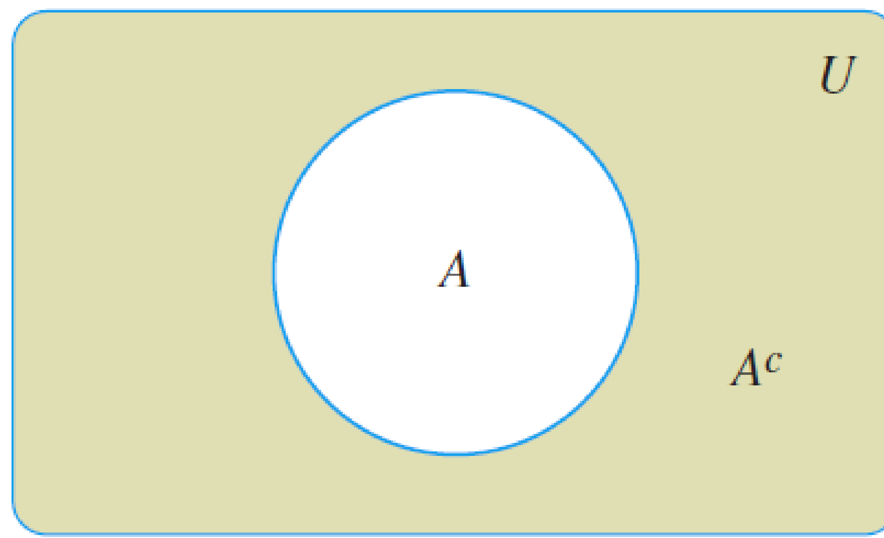


FIGURE 4
Set complementation

- **<u>Set Union</u>:** The set of all elements that belong to A **or** B

$$A \cup B = \{x \mid x \in A \text{ or } x \in B \text{ or both}\}$$

- EX. A = {a, b, c} B = {a, c, d}

$$A \cup B = \{a, b, c, d\}$$

- **<u>Set Intersection</u>**: the set of all elements that are common to A **and** B.

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

- EX. A = {a, b, c} B = {a, c, d}

$$A \cap B = \{a, c\}$$

- EX. A = {1,3,5,7,9} B = {2,4,6,8}

$$A \cap B = \varnothing$$

When two sets have just the empty set in common, they are called **<u>disjoint</u>**.

- EX. Given the sets:

U = {m, a, t, h, x, y}

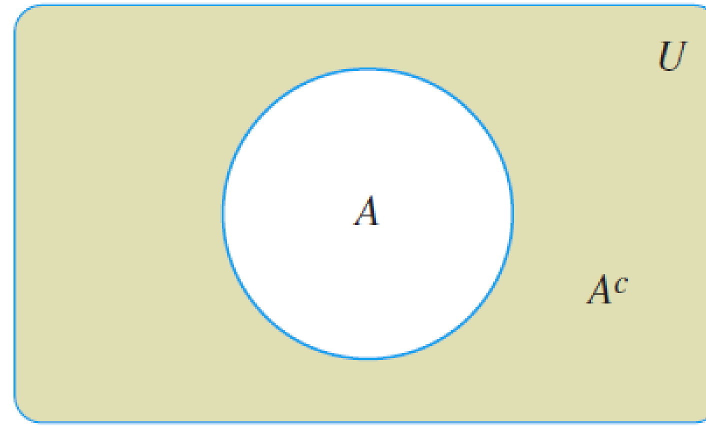
A = {m, a, t, h}

B = {a, h, x, y}

$$A \cup B = \{m, a, h, t, x, y\}$$

$$A \cap B = \{a, h\}$$

- **Complement of a Set:** If U is a universal set and A is a subset of U, then the set of all elements in U that are not in A is called the complement of A, written A^c.



EX. Find a U and an A such that

$A^C = \{1, 2, 4, 8\}$

Set Complementation: If U is the universal set and A is a subset of U, then

$$U^C = \varnothing \qquad \varnothing^C = U$$

$$\left(A^C\right)^C = A \qquad A \cap A^C = \varnothing \qquad A \cup A^C = U$$

DO THESE MAKE SENSE???

Some Examples

$x \in U$, where U is GHHS students.

S = {x | x is a Senior},

F = {x | x is a Female},

E = {x | x is 18 years old at GHHS}

1. Write, using notation, the set of all 18 year olds at GHHS who are not seniors.

$$E \cap S^C$$

2. Write what this means in words:

$F^c \cup S$ A male or a senior

Sometimes it is important to find the *number* of elements in a set or combination of sets.

Such problems are called COUNTING PROBLEMS and constitute a field of study known as COMBINATORICS.

The number of elements in a set is denoted:

n(A)

Consider these sets: $A = \{1,2,3,\ldots 20\}$

$B = \{ a, b \}$ $C = \{ 8 \}$ $D = \{ \ \}$

So, $n(A) = 20$ $n(B) = 2$ $n(C) = 1$ $n(D) = 0$

EX 1: $A = \{ a, c, d \}$ $B = \{ b, e, f, g \}$

$n(A \cup B) = n(A) + n(B) = 7$

EX 2: $A = \{ a, b, c, d, e \}$ $B = \{ b, d, f, g\}$

$A \cup B = \{ a, b, c, d, e, f, g \}$ which has 7 elements, but….

$n(A) + n(B) = 5 + 4 = 9$. **Why?**

Can you use math notation to write a general rule to find the number of elements in the union of two sets?

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

GENERAL ADDITION CASE

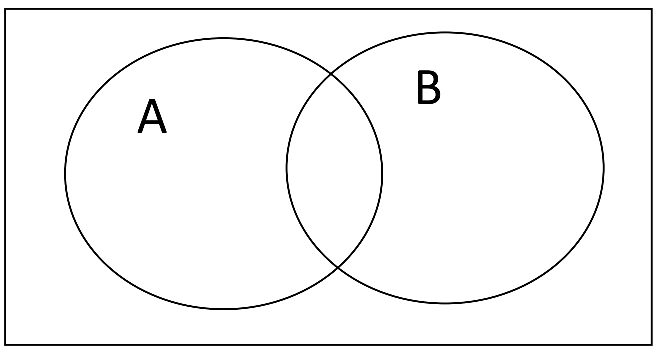
$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Reminder:

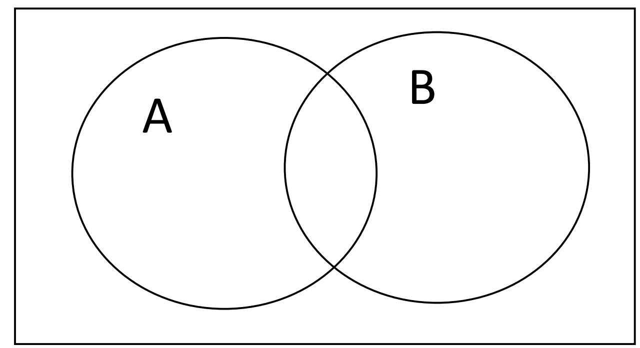
VENN DIAGRAMS: DIAGRAMS THAT SHOW ALL POSSIBLE LOGICAL RELATIONS BETWEEN A FINITE COLLECTION OF SETS

Venn Diagram Examples: Given sets A and B, shade the following:

- $A \cup B$



$(A \cap B)^c$



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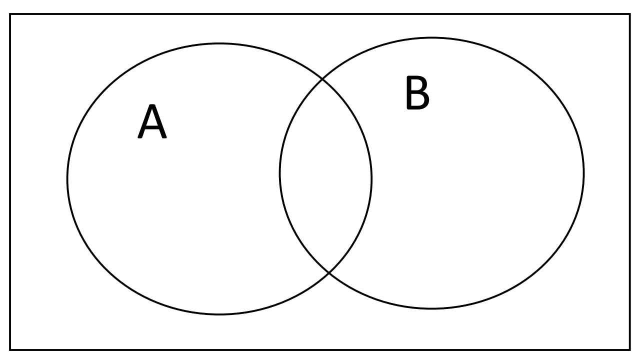

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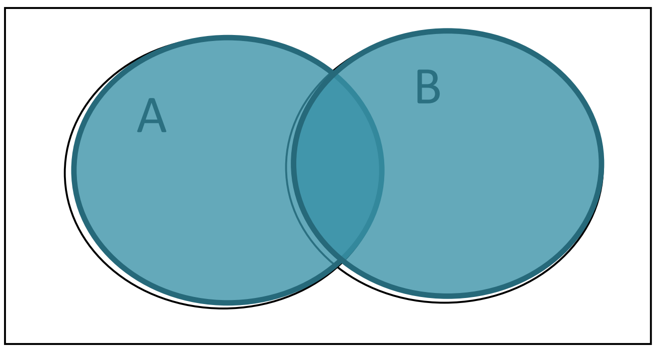

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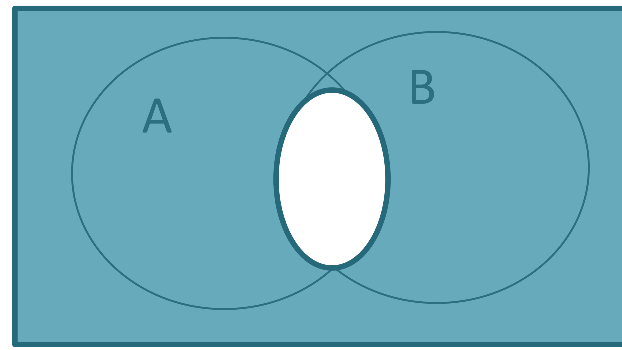
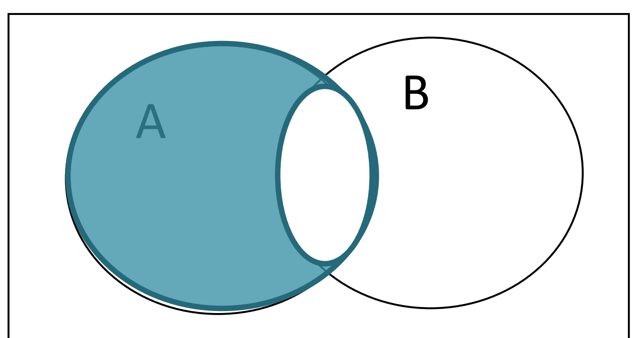


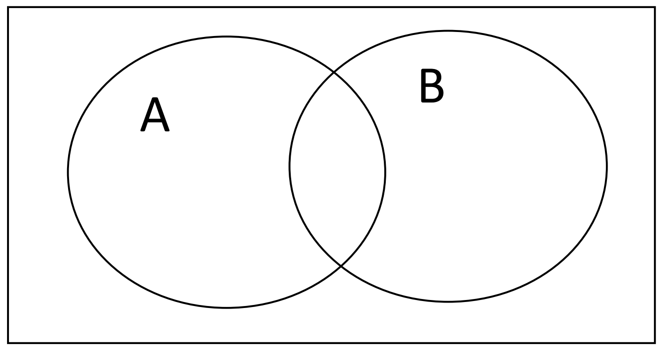
- $A \cap B^c$



DeMorgan's Law

- Let A & B be sets, then

$$(A \cup B)^C = A^C \cap B^C$$



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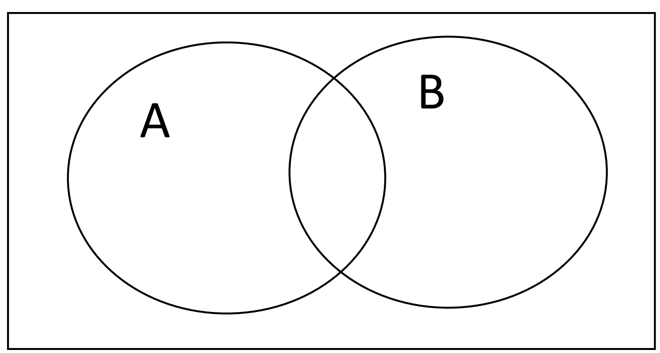

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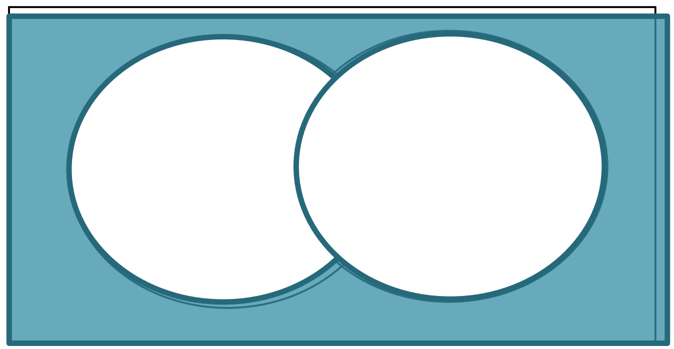

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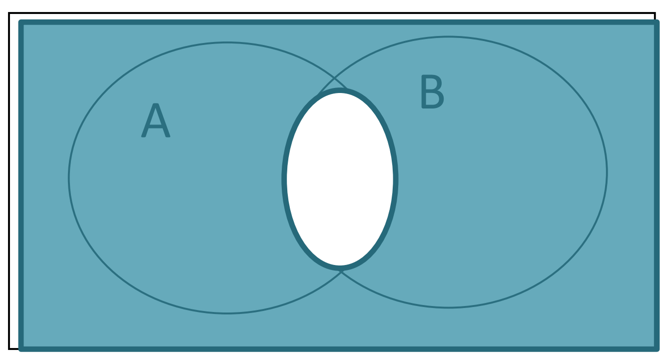
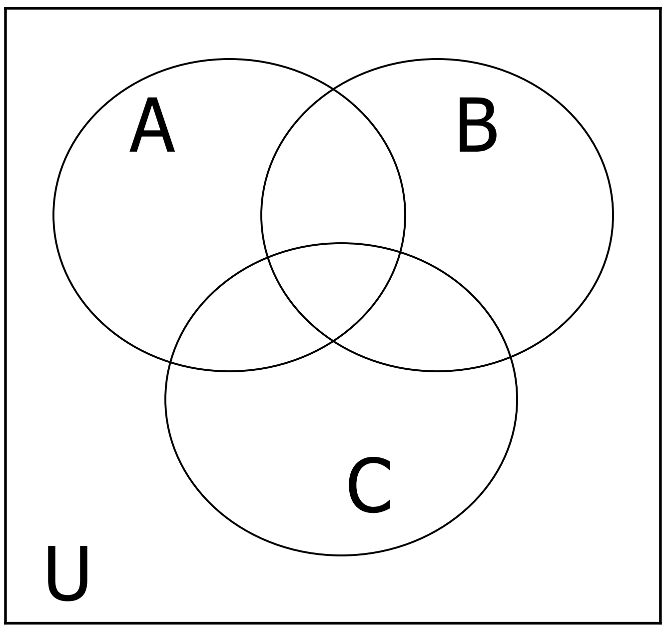


What would these look like??

$A \cap B \cap C$



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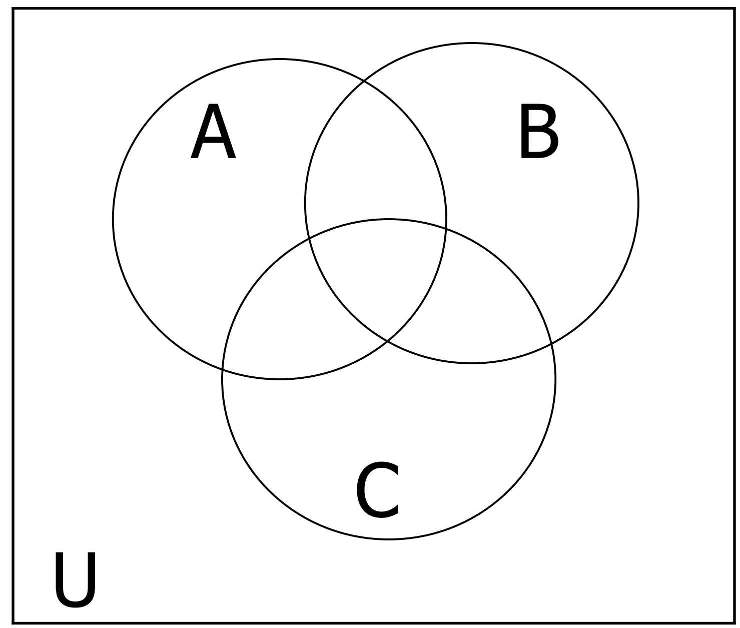

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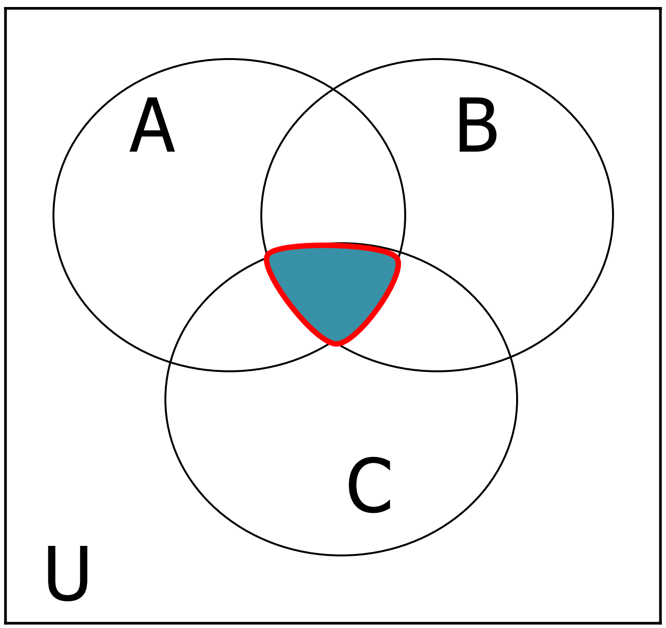

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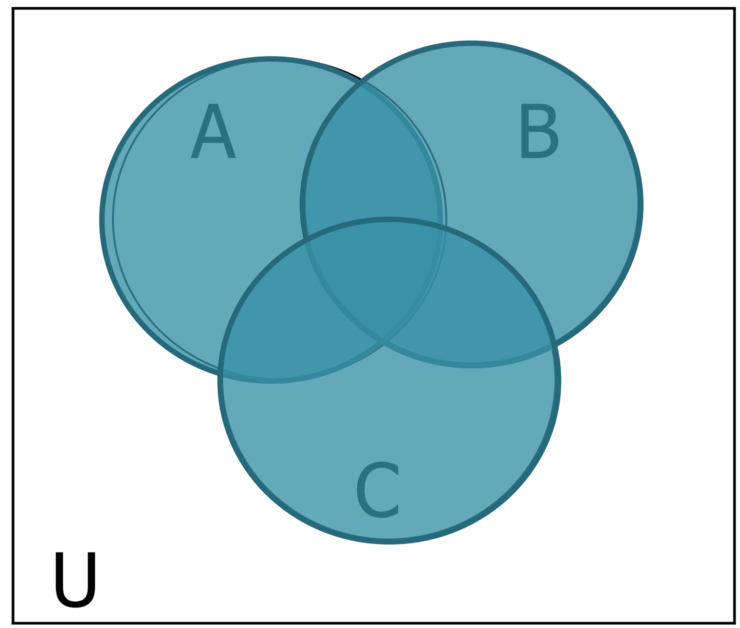


Consider the Universal Set consisting of the integers between 1 and 8. Given A={2,3,5,7}, B={2,4,6,8}, C={6} and D={3,4,5,6}, use Venn diagrams to help find the following.

$A \cap D$

$B \cup D$

A^C

$C^C \cup B$

$B^C \cap D$

Consider the Universal Set consisting of the integers between 1 and 8. Given A={2,3,5,7}, B={2,4,6,8}, C={6} and D={3,4,5,6}, use Venn diagrams to help find the following.

$A \cap D = \{3, 5\}$

$B \cup D = \{2, 3, 4, 5, 6, 8\}$

$A^C = \{1, 4, 6, 8\}$

$C^C \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$B^C \cap D = \{3, 5\}$

Homework

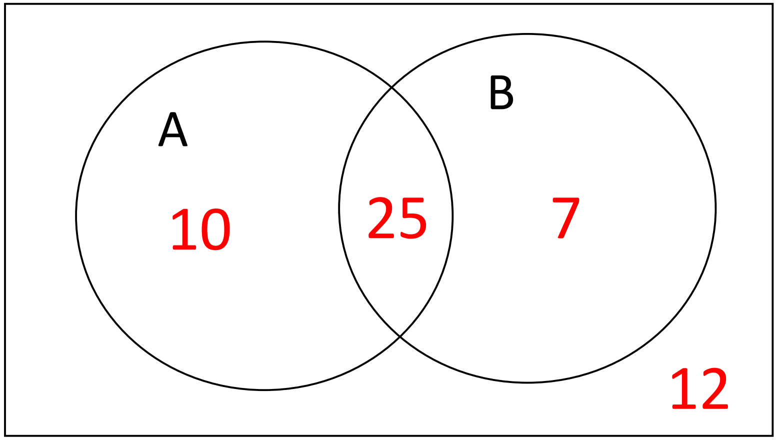
Packet p. 1-2

Next slides….

Moved from Day 2 for Fall ‘17

Use Venn diagrams to solve the following.

Suppose there are a total of 54 people in which 35 of them are type A, 32 of them are type B, while 7 are B, but not A.



How many are in A, but not B?

A=10

How many are in A and B?

A and B=25

Example:

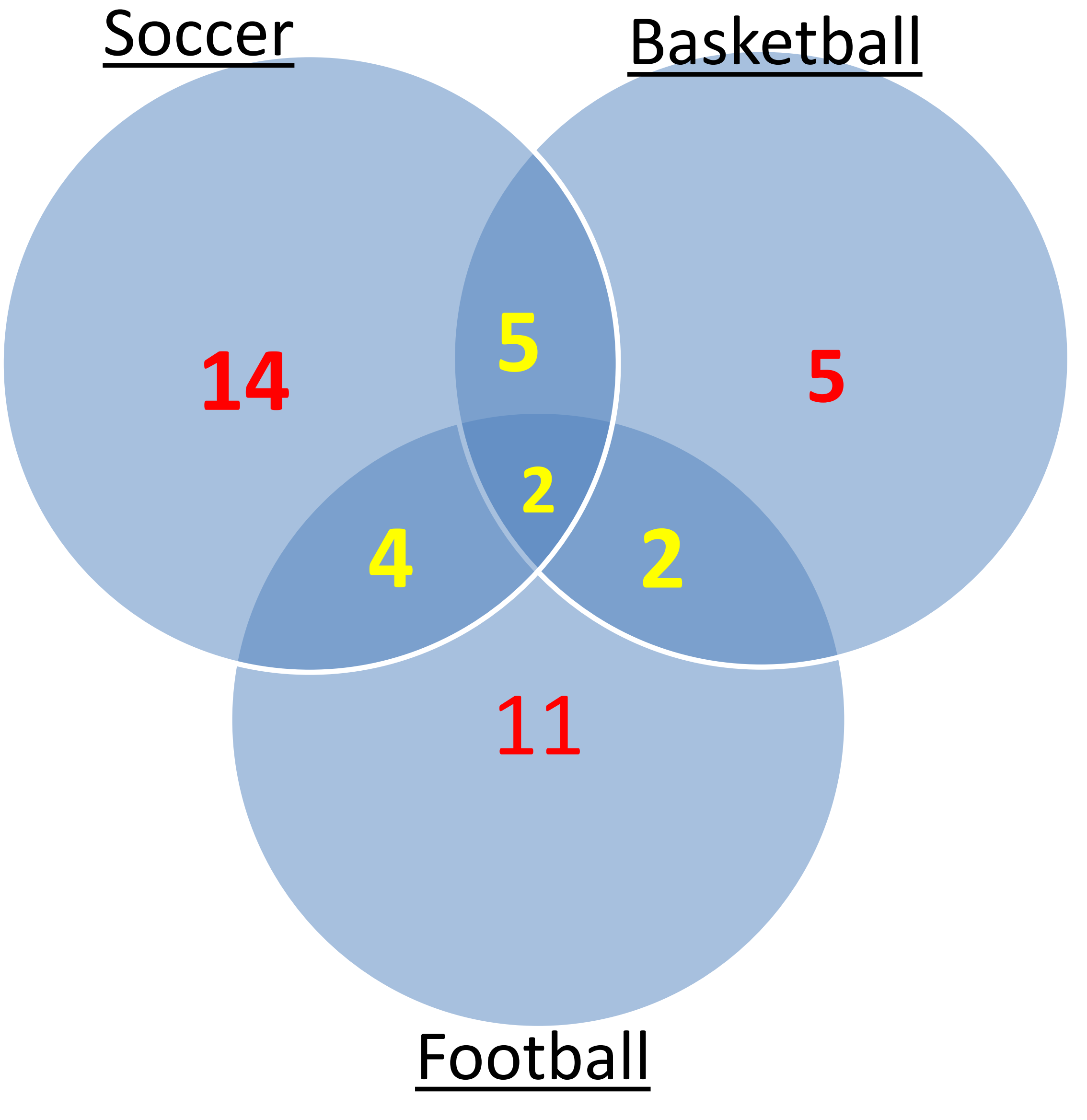
In a club of varsity athletes, 25 athletes played soccer, 14 athletes played basketball, 19 athletes played football, 7 athletes played soccer and basketball 6 athletes played soccer and football, 4 athletes played basketball and football and 2 athletes played all three sports.

Draw a Venn Diagram and answer the questions.

1. How many athletes played soccer, but not basketball or football? 14

2. How many athletes played soccer and basketball, but not football? 5

3. How many athletes played just one of the three sports? 30


Soccer
Basketball
14
5
5
2
4
2
11
Football

Example: YOU TRY!

- A survey of 300 workers yielded the following information: 231 belonged to the Teamsters Union, and 195 were Democrats. If 172 of the Teamsters were Democrats, how many workers were in the following situations?

A. Belonged to the Teamsters or were Democrats

254

B. Belonged to the Teamsters but were not Democrats

59

C. Were Democrats but did not belong to the Teamsters

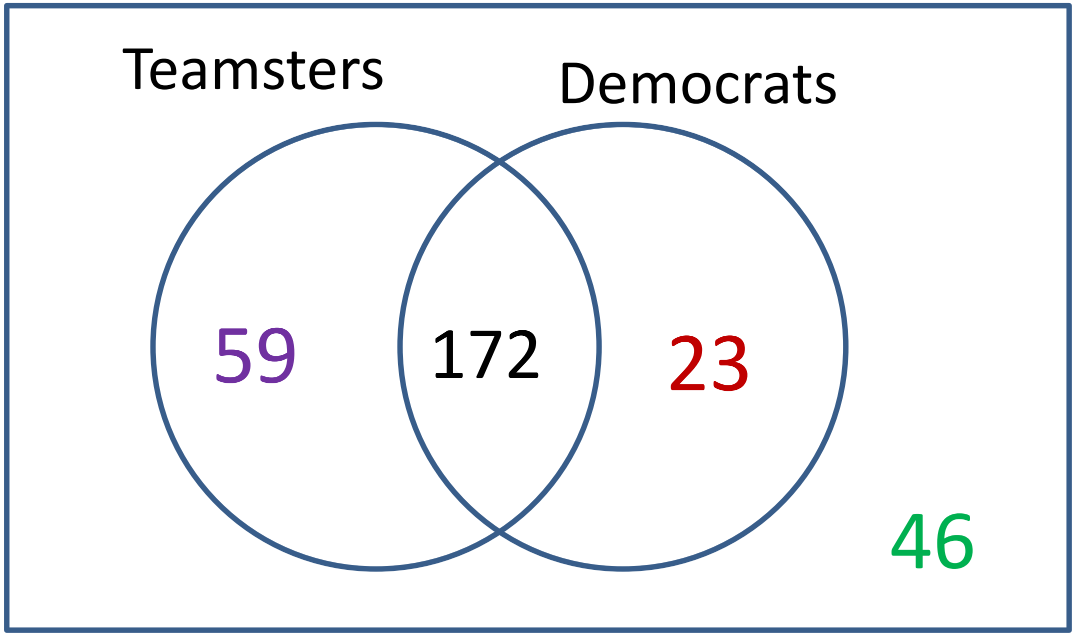
23

D. Neither belonged to the Teamsters nor were Democrats.

46

Let us define T = The event that a worker belongs to the Teamsters, and D = the event that a worker is a Democrat. Note that $n(U) = 300$.

We were given the fact that 172 workers were both in the Teamsters and a Democrat, that is $n(T \cap D)=172$



Those who are only Democrats and do not belong to the Teamsters are $n(D) - n(T \cap D) = 195 - 172 = 23$.

Those who are only Teamsters but not Democrats are $n(T) - n(T \cap D) = 231 - 172 = 59$.

Example: You try!

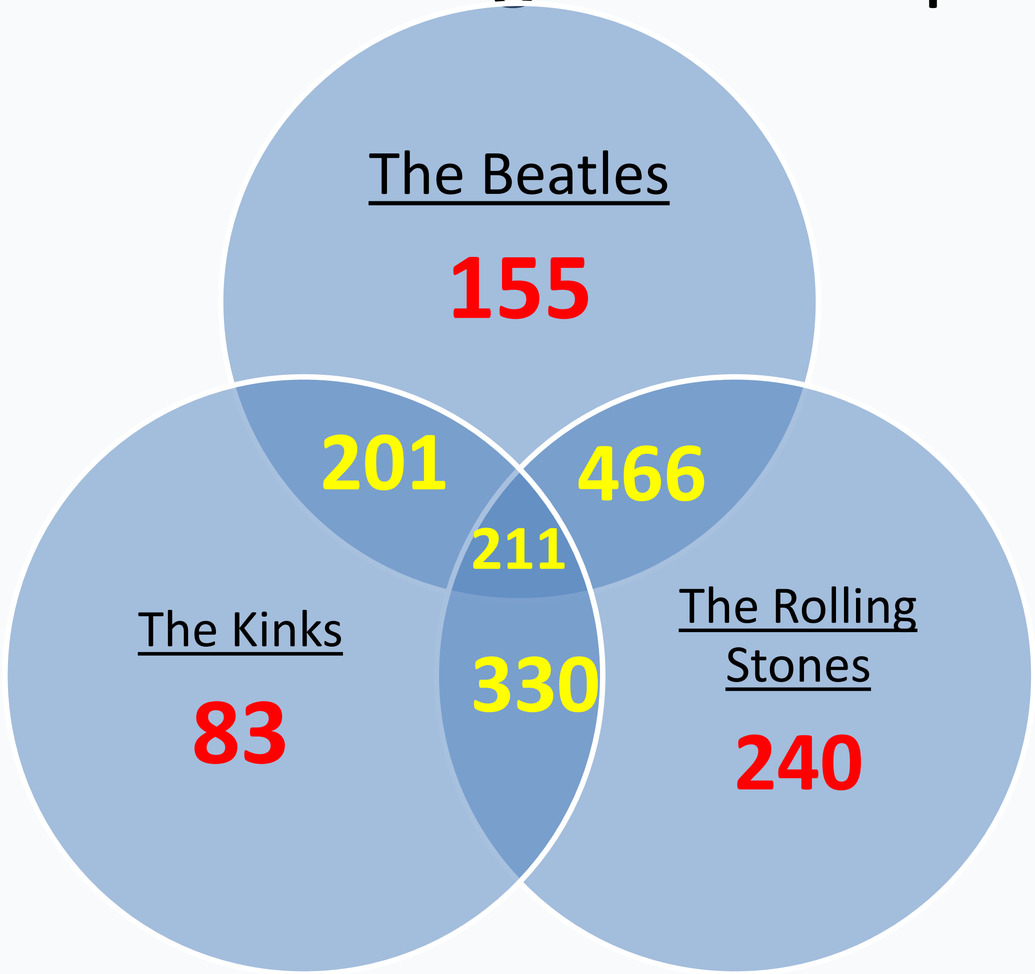
- Way back in 1965, The Beatles, The Kinks, and The Rolling Stones toured the USA. A large group of teenagers were surveyed and the following information was obtained:

- 825 saw The Kinks,
- 1033 saw The Beatles,
- 1247 saw The Rolling Stones,
- 211 saw all three, 514 saw none,
- 240 saw only The Rolling Stones,
- 677 saw The Rolling Stones and The Beatles,
- 201 saw The Beatles and The Kinks but not The Rolling Stones.

A. What percent of the teenagers saw at least one band?

B. What percent of the teenagers saw exactly one band?

Venn Diagram Example 2



A. What percent of the teenagers saw at least one band?

1686/2200 = 76.6 %

B. What percent of the teenagers saw exactly one band?

478/2200 = 21.7 %

Explanation

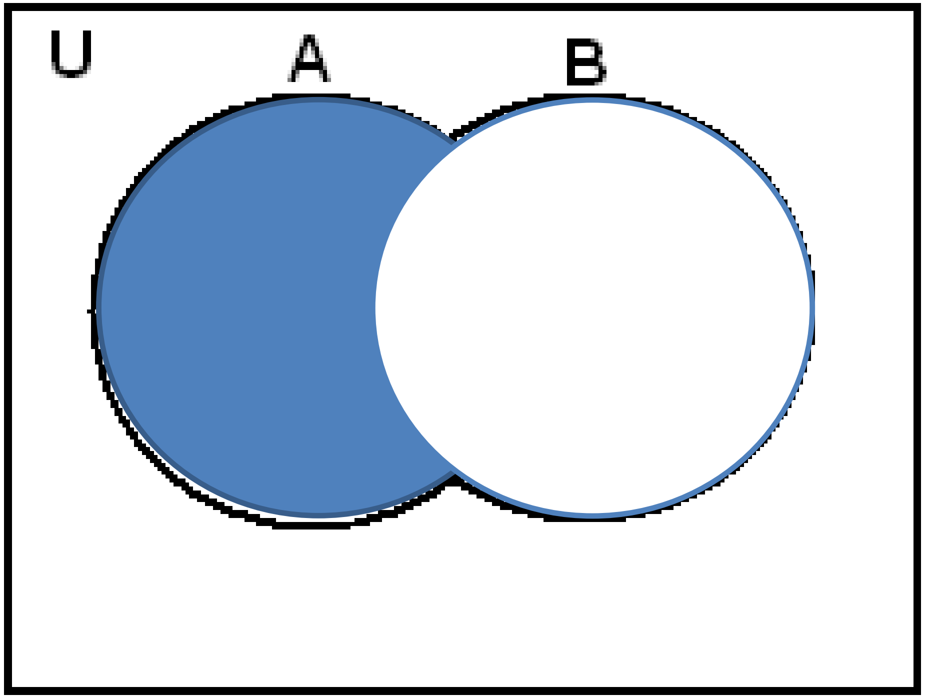
- Begin labeling the diagram with the innermost overlap. 211 saw all three.
- Note also that the region outside of the circle contains 514. These are teens who saw no band.
- Continue using the given info: 240 saw only the Rolling Stones.
- 201 saw The Beatles and The Kinks but not The Rolling Stones.
- 677 saw The Beatles and The Rolling Stones. Now that region already has 211 people in there. Take 677-211 and that will give 466 people who saw The Beatles and the Rolling Stones but not The Kinks.
- Now we can find the people who only saw The Beatles.
- Take 1033 and subtract (201+211+466). This gives 155 people who only saw The Beatles.
- To find the number of people who saw The Rolling Stones and The Kinks but not the Beatles, take 1247 – (211+466+240)=330.
- To find the number of people who only saw The Kinks 825 – (211+ 201+33) = 83.
- The total number of people surveyed is 83+201+211+330+240+446+155+514=2200.
- Now answer question A: The people who saw at least one band is (2200-514)/2200=76.6%
- Question B: The people who saw only one band is (83+155+240)/2200=21.7%.

Whiteboard Practice

- Get a whiteboard, marker and eraser!

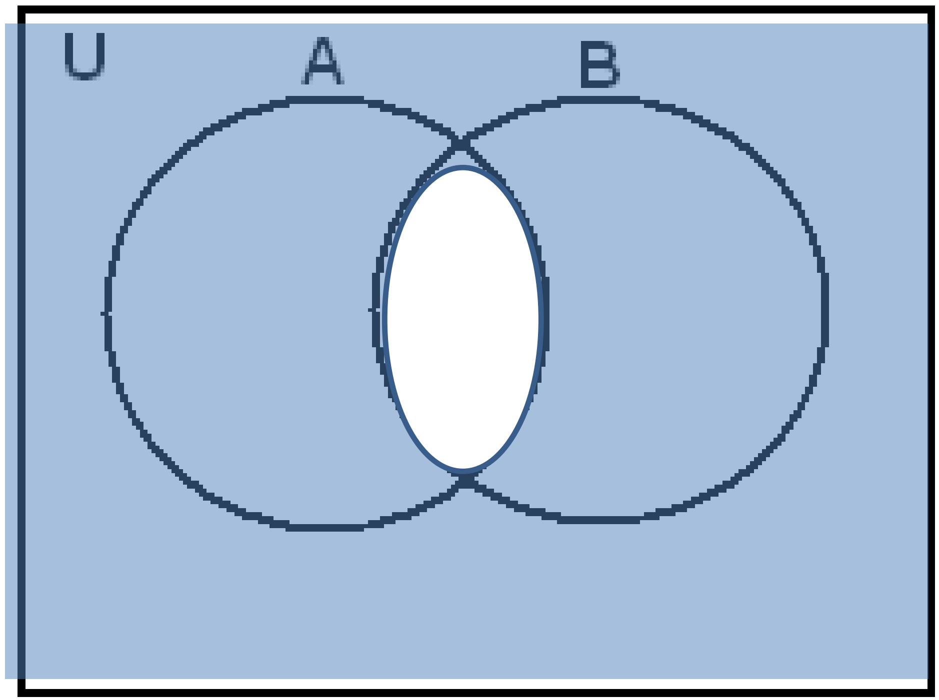
Example 1: Shade the Venn Diagram

$$A \cap B^c$$



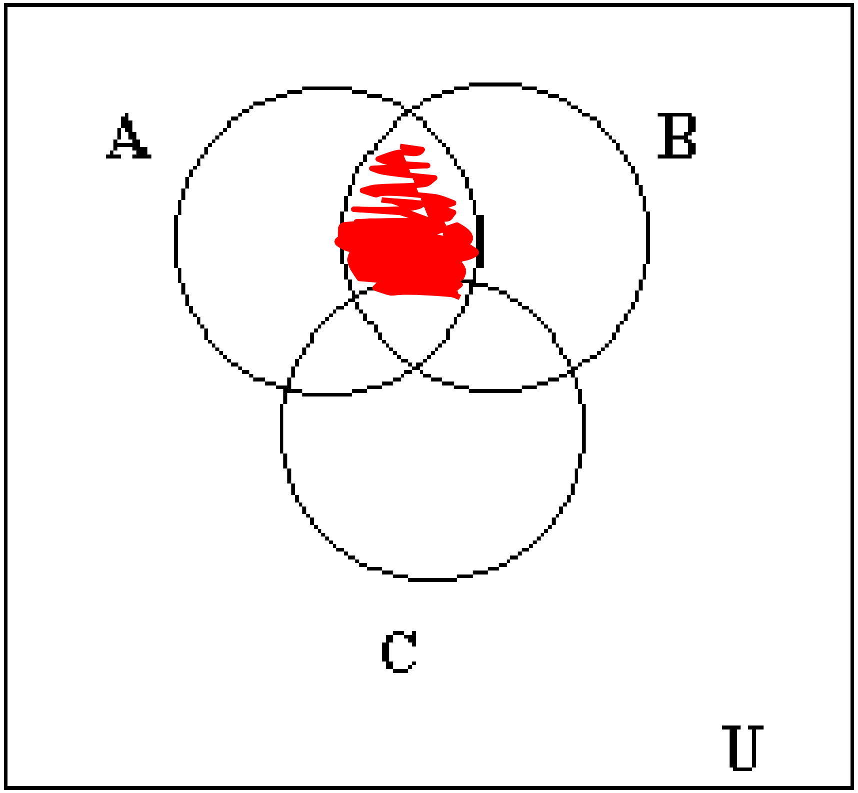
Example 2: Shade the Venn Diagram

$$A^c \cup B^c$$



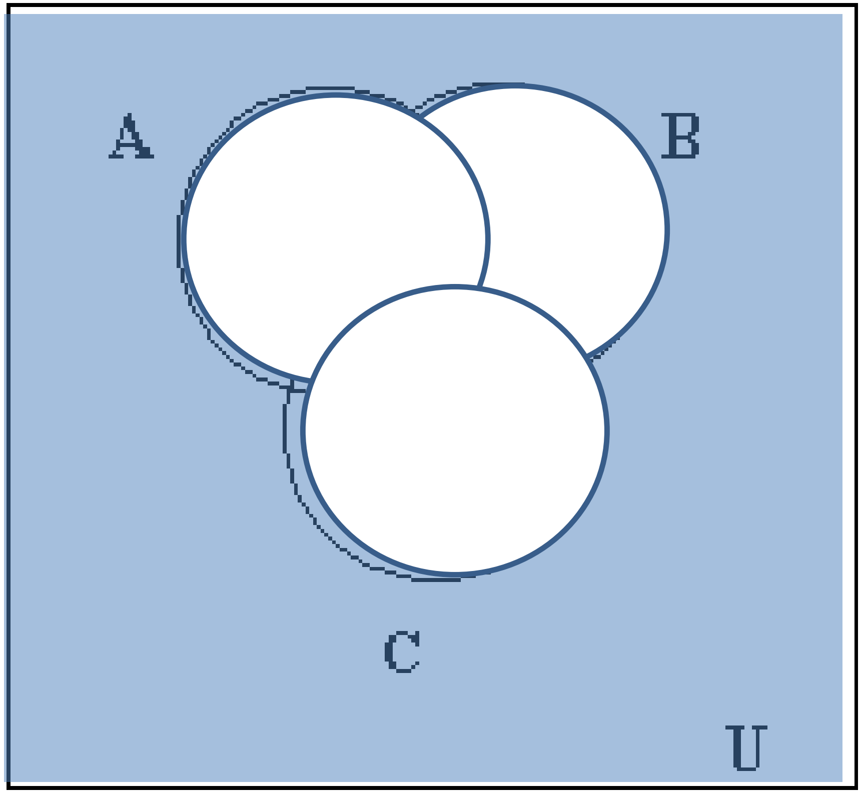
Example 3:

$$A \cap B \cap C^c$$



Example 4:

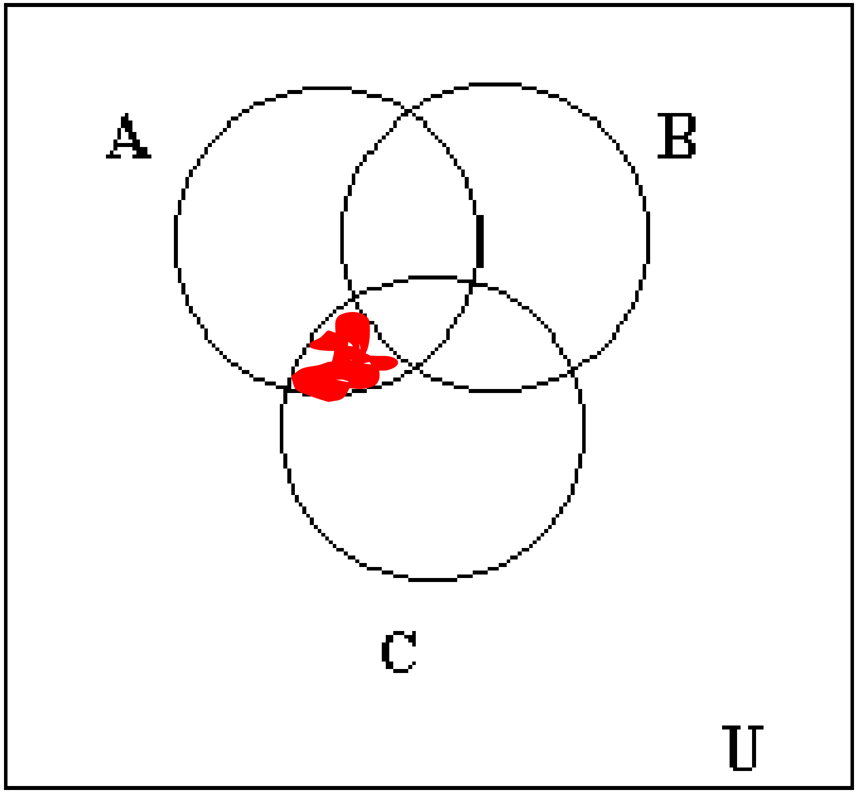
$$A^c \cap B^c \cap C^c$$



Example 5:

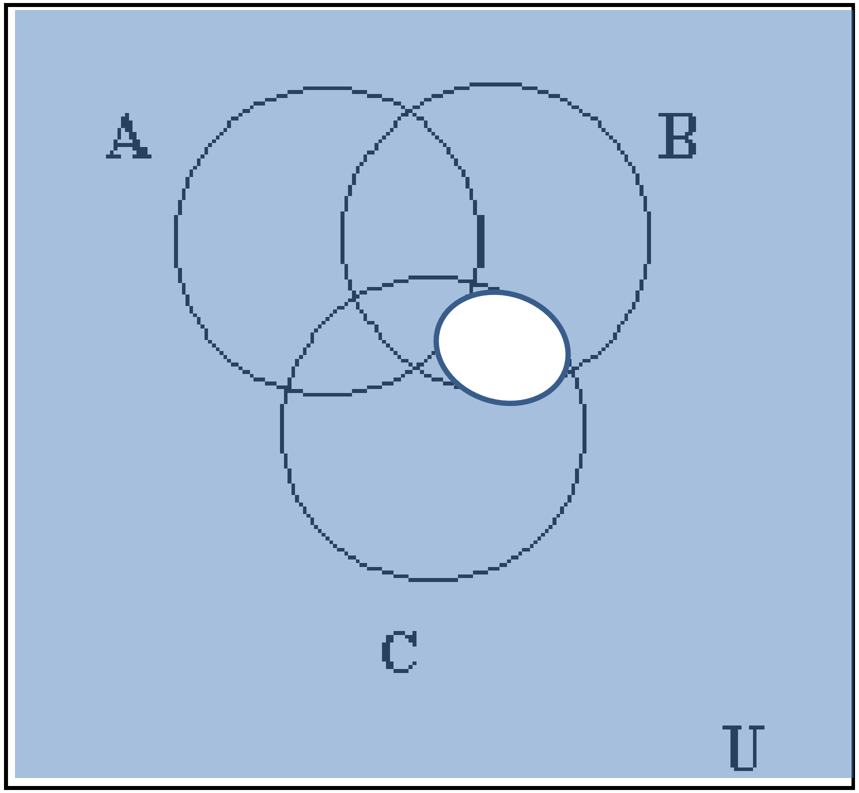
Draw and shade the Venn Diagram.

$$A \cap C \cap B^c$$



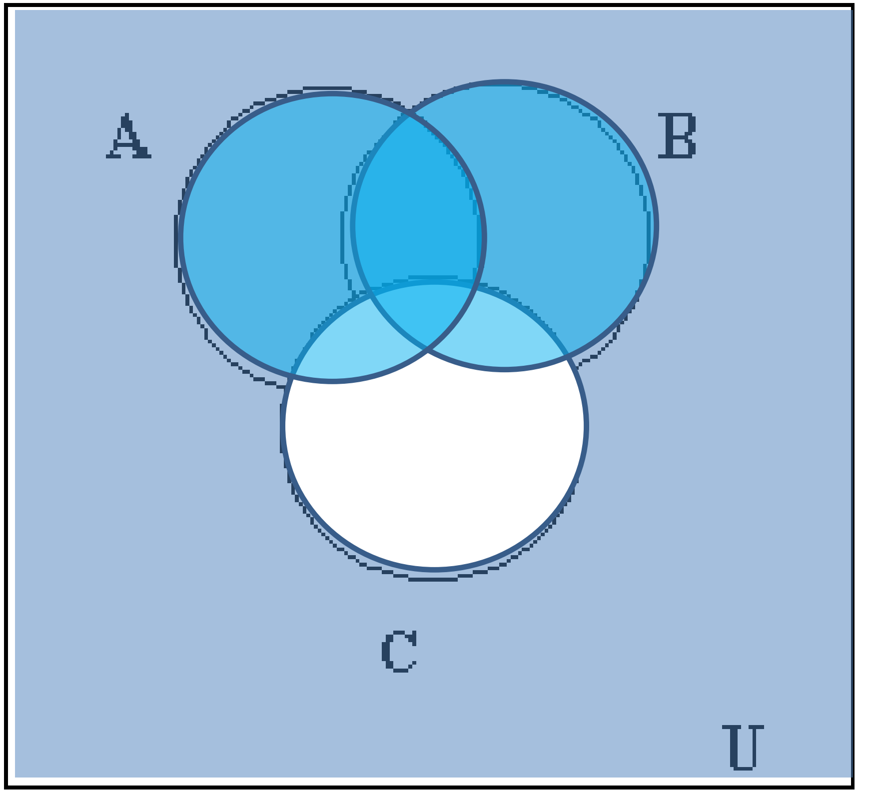
Example 6:

$$A \cup \left(B \cap C\right)^{c}$$



Example 7:

$$A \cup B \cup C^c$$



Example 8:

$$A^c \cap B \cap C$$