Unit 1 Day 0

Set Operations & Venn Diagrams

SET OPERATIONS & VENN DIAGRAMS

Unit 1: Sections 6.1-6.2

In life we like to put objects, people, animals, etc. into groups, categories, or "sets" as we call them in math.

Meet your table partners and figure out what sets you have in common. Be creative!

Yes, you are going to share these with the class!

I. Definitions

- SET: A collection of well-defined and distinct objects
- ELEMENT: Any one of the distinct objects that make up that set

A set must be well-defined in that if we are given an object, we should be able to determine whether or not it belongs in the collection.

EX. $A = \{w, a, r, d\}$ → **roster notation**

Sets are often named with capital letters	Order does not matter, no duplicates

I. Definitions

A set must be well-defined in that if we are given an object, we should be able to determine whether or not it belongs in the collection.

EX. $A = \{w, a, r, d\}$ → **roster notation**

Sets are often named with capital letters

Order does not matter, no duplicates

Notation: $w \in A$ w is an element of set A

EX. $A = \{1, 2, 3, 4, 5\}$

$6 \notin A \rightarrow$ use to show 6 is not an element of A

$3 \in A \rightarrow$ use to show 3 is an element of A

I. Definitions

- <u>SUBSET</u>: If every element of set A is also an element of set B, then A is a subset of B, and is written $A \subseteq B$.

  EX. A = {r, d} **B = {r, a, w, d, e, t}**

  Every element in A is also in B so $A \subseteq B$.

- <u>EMPTY SET (NULL SET)</u>:
  a set containing no elements.
  The empty set is a subset of every set.
  It is denoted $\varnothing$.
  The number 0 is not the same as the empty set!

  EX. $\varnothing \subseteq B$

- **Set Equality:** Two sets A & B are equal if they have exactly the same elements.

EX. **A = {a, w, r, d}** **B = {d, r, a, w}**

Every element in A is in B, and every element in B is in A.

$$A = B$$

* **Is A a subset of B?** Yes! $\mathbf{A \subseteq B}$

* **Is B a subset of A?** $\mathbf{B \subseteq A}$

Which of the following is equal to sets A and B?

i. {x| x is a letter of the word **raw**}

ii. {x| x is a letter of the word **ward**}

iii. {x| x is a letter of the word **award**}

ii and iii!

- **<u>Proper Subset</u>: If A and B are sets such that $A \subseteq B$, but $A \neq B$, then A is a proper subset of B.**

EX. $A = \{2, 4, 6\}$ $B = \{1, 2, 3, 4, 5, 6\}$ → $A \subset B$

(A is a proper subset of B)

Think A is properly "smaller" than B!

Notice the notation.

*What about the last example? Is $A \subset B$?

A = {w, a, r, d} **B= {d, r, a, w}**

Nope, $A \not\subset B$ (because A = B)!

EX. Let A = {a, e, i, o, u} B = {a, i, o, e, u}

C = {a, e, i, o} D = {a, e, i, o, x}

- TRUE OR FALSE? (on the board…)

$A = B$ $D \subsetneq B$ $B \subseteq A$

$C \subseteq B$ $D \subsetneq A$ $C \subset B$

$C \subseteq A$ $A \subseteq B$ $C \subset A$

ALL TRUE ☺

What about $\mathrm{B} \subset A$? **FALSE, THEY ARE EQUAL!**

- **Universal Set**: The set of all elements of interest in a particular discussion is called the universal set.

* **All sets considered in a problem are subsets of the universal set.***

- EX. List all the subsets of the set $A = \{a, b, c\}$

$\varnothing, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}$

Remember! By definition, the empty set $\varnothing$ is a subset of all sets!

- **Set builder notation:** A rule that describes the definite property (properties) an object x must satisfy to be part of the set.

- EX. $A = \{x \mid x \text{ is an even integer}\}$
  "*x* such that *x* is an even integer"

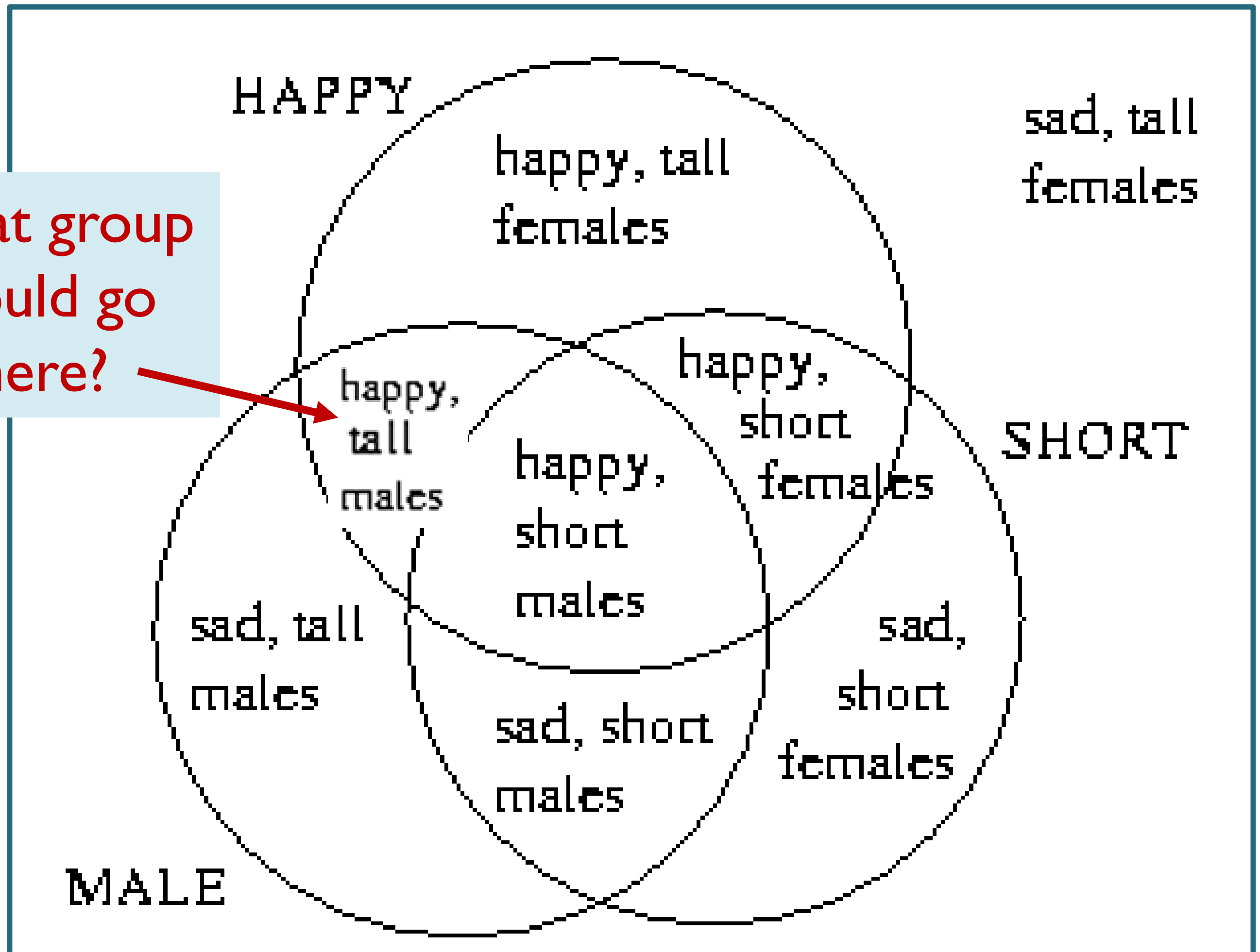
- EX. The set B of all letters of the alphabet

  Roster notation: $B = \{a, b, c, \ldots, x, y, z\}$

  Set-builder notation: $B = \{x \mid x \text{ is a letter of the English alphabet}\}$

Definitions continued

- VENN DIAGRAMS: Diagrams that show all possible logical relations between a finite collection of sets



What group would go here?

Set Operations: The ways in which sets can be combined to yield other sets.

- UNION: The union of two sets is the set obtained by combining the members of each.
- INTERSECTION: The intersection of two sets is the set of elements common to A and B.
- COMPLEMENT: a **complement** of a set *A* refers to elements not in A.

More about these terms on the next slides ->

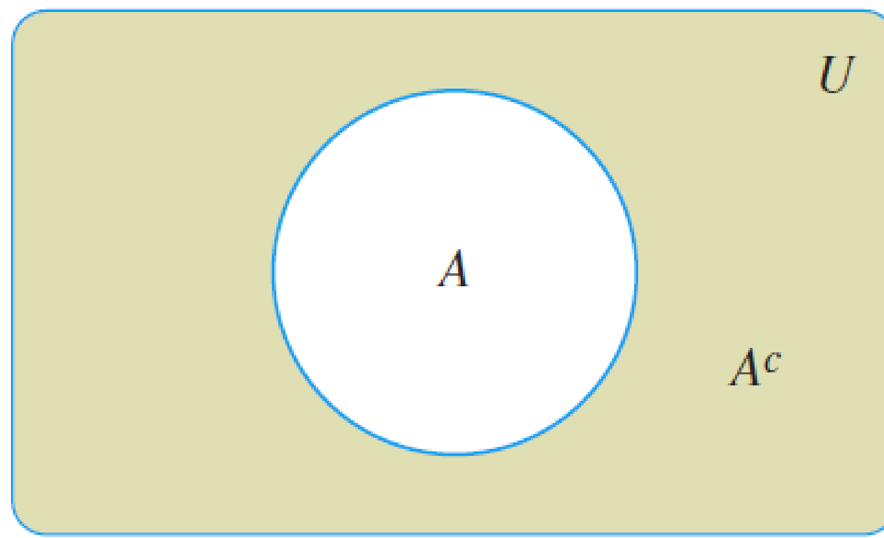


FIGURE 4
Set complementation

- **<u>Set Union</u>:** The set of all elements that belong to A **or** B

$$A \cup B = \{x \mid x \in A \text{ or } x \in B \text{ or both}\}$$

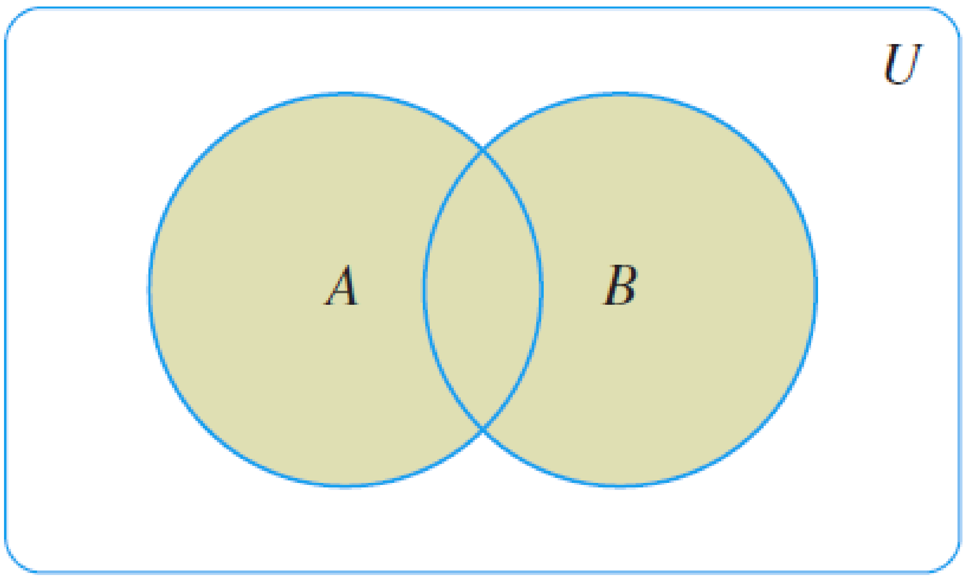


FIGURE 2
Set union $A \cup B$

EX: Find A U B
A = {a, b, c}
B = {a, c, d}

$$A \cup B =$$

{a, b, c, d}

- **<u>Set Intersection</u>**: the set of all elements that are common to A **and** B.

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

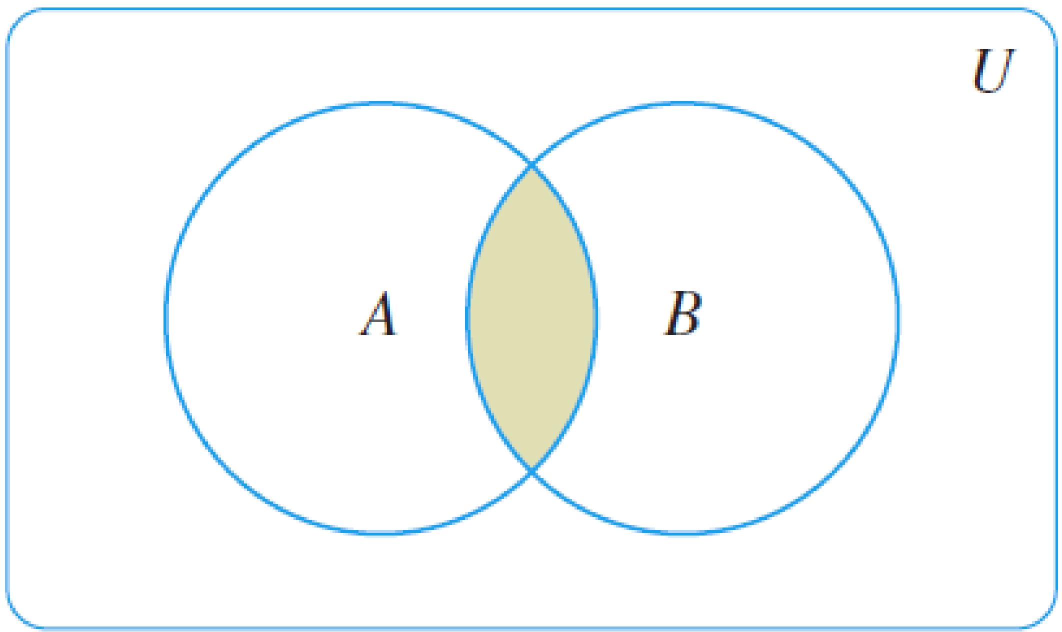


FIGURE 3
Set intersection $A \cap B$

EX. Find $A \cap B$ if

A = {a, b, c}
B = {a, c, d}

$A \cap B =$

{a, c}

When two sets have just the empty set in common, they are called **disjoint**.

- EX. A = {1,3,5,7,9} B = {2,4,6,8}

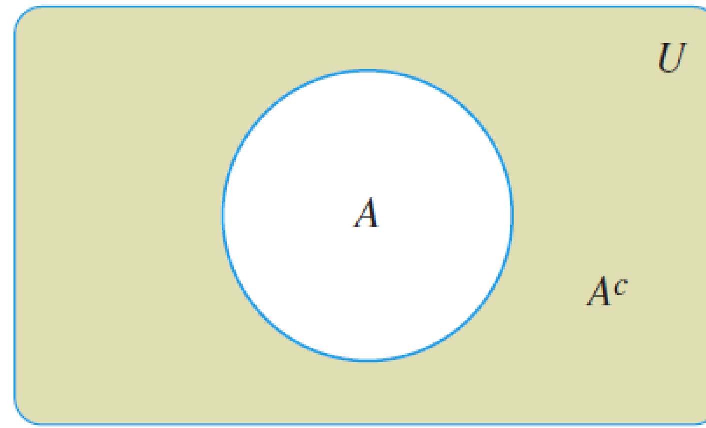
  a) Find $A \cap B$

  b) What term describes sets A and B?

a) $A \cap B = \varnothing$

b) Sets A and B are ***disjoint*** sets because they do not intersect.

- **<u>Complement of a Set</u>:** If U is a universal set and A is a subset of U, then the set of all elements in U that are not in A is called the complement of A, written A^c.
  Think: A^c = "not A"



EX. Find a U and an A such that
$A^C = \{1, 2, 4, 8\}$

<u>Set Complementation</u>: If *U* is the universal set and *A* is a subset of *U*, then

$$U^C = \varnothing \qquad \varnothing^C = U$$

$$\left(A^C\right)^C = A \qquad A \cap A^C = \varnothing \qquad A \cup A^C = U$$

DO THESE MAKE SENSE???

- You Try!

EX. Given the sets:

U = {m, a, t, h, x, y}

A = {m, a, t, h}

B = {a, h, x, y}

Find $A \cup B =$ {m, a, t, h, x, y}

$A \cap B =$ {a, h}

$A^c =$ {x, y}

Some Examples

$x \in U$, where U is GHHS students.

S = {x | x is a Senior},
F = {x | x is a Female},
E = {x | x is 18 years old at GHHS}

1. Write, using notation, the set of all 18 year olds at GHHS who are not seniors.

$$E \cap S^C$$

2. Write what this means in words:

$F^c \cup S$ A male or a senior

3. Would a female senior be an element of the set in #2? Yes because she is a senior. It's **not an intersection** so she doesn't have to be male.

Sometimes it is important to find the ***number*** of elements in a set or combination of sets.

Such problems are called COUNTING PROBLEMS and constitute a field of study known as COMBINATORICS.

The number of elements in a set A is denoted: **n(A)**

Consider these sets: A = {1, 2, 3, . . . 20}

B = { a, b } C = { 8 } D = { }

So, n(A) = 20 n(B) = 2 n(C) = 1 n(D) = 0

EX 1: $A = \{ a, c, d \}$ $B = \{ b, e, f, g \}$

$$n(A \cup B) = n(A) + n(B) = 7$$

EX 2: $A = \{ a, b, c, d, e \}$ $B = \{ b, d, f, g\}$

$A \cup B = \{ a, b, c, d, e, f, g \}$ which has 7 elements, but….

$n(A) + n(B) = 5 + 4 = 9$, NOT 7. **Why?**

Sets A and B have elements in common, so we can't simply add their values to find their union.

Can you use math notation to write a general rule to find the number of elements in the union of two sets?

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

GENERAL ADDITION CASE

$$n\left(A\cup B\right)=n\left(A\right)+n\left(B\right)-n\left(A\cap B\right)$$

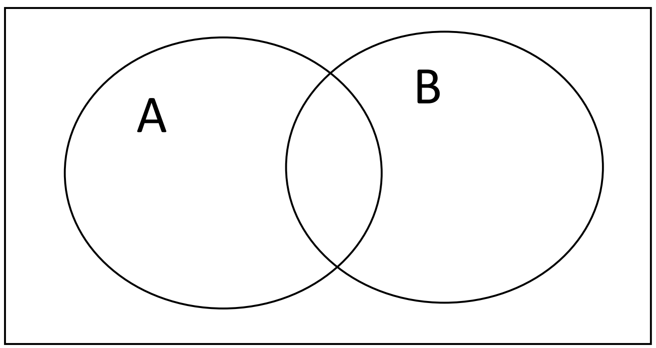
Reminder:

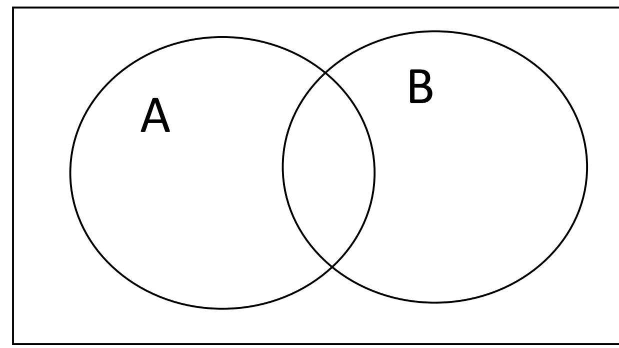
VENN DIAGRAMS: DIAGRAMS THAT SHOW ALL POSSIBLE LOGICAL RELATIONS BETWEEN A FINITE COLLECTION OF SETS

Venn Diagram Examples: Given sets A and B, shade the following:

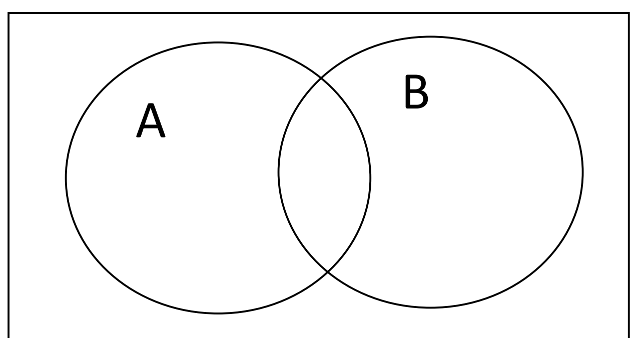
- $A \cup B$

$(A \cap B)^c$





- $A \cap B^c$



Venn Diagram Examples: Given sets A and B, shade the following:

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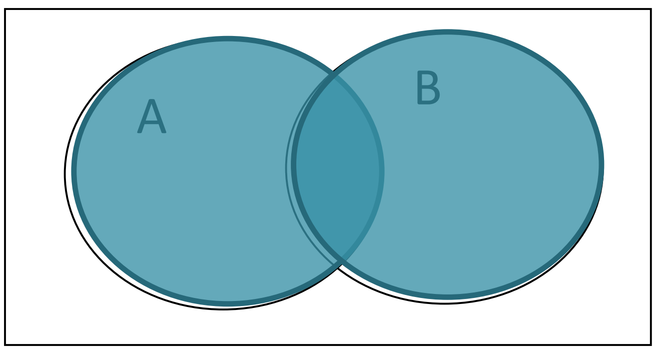

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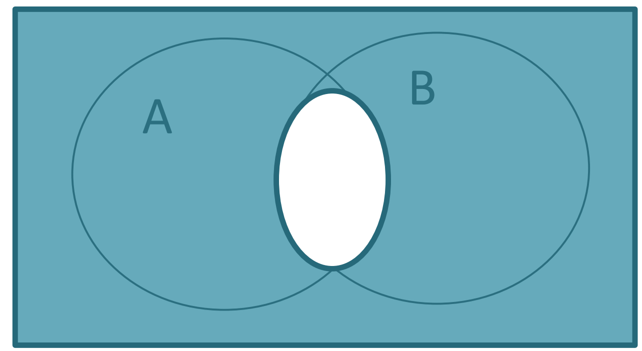
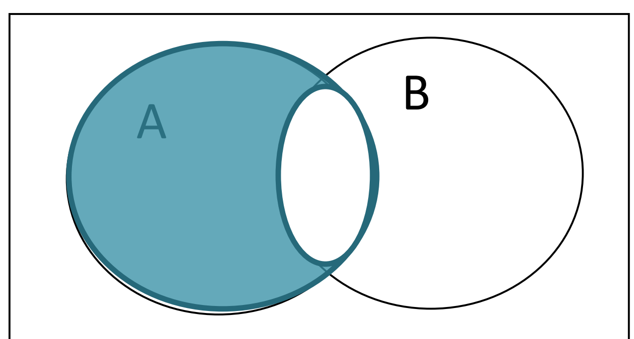


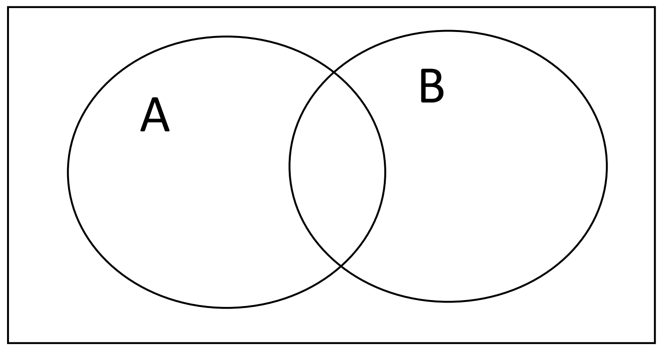
- $A \cap B^c$



DeMorgan's Law

- Let A & B be sets, then

$$(A \cup B)^C = A^C \cap B^C$$



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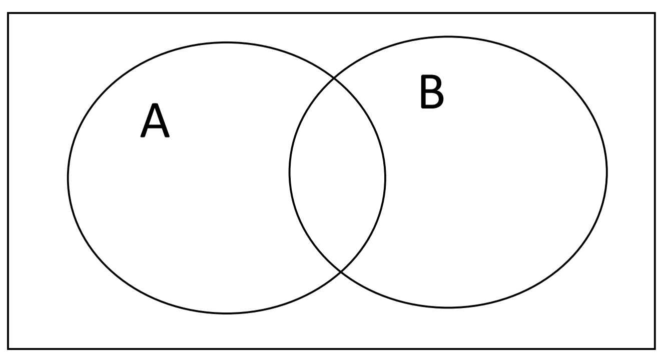

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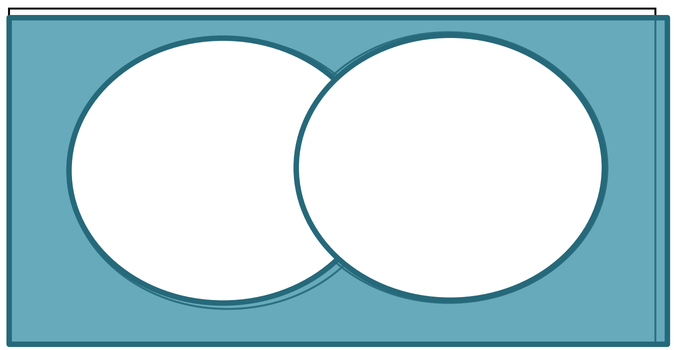

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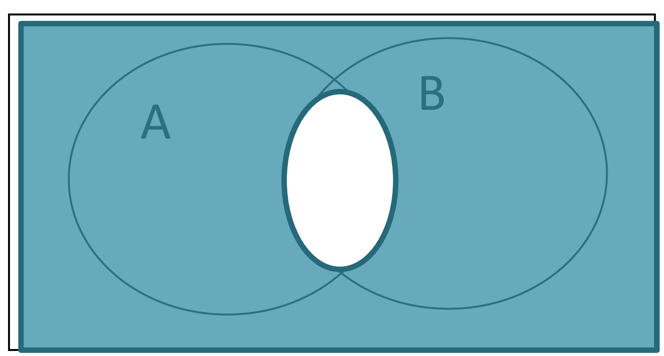
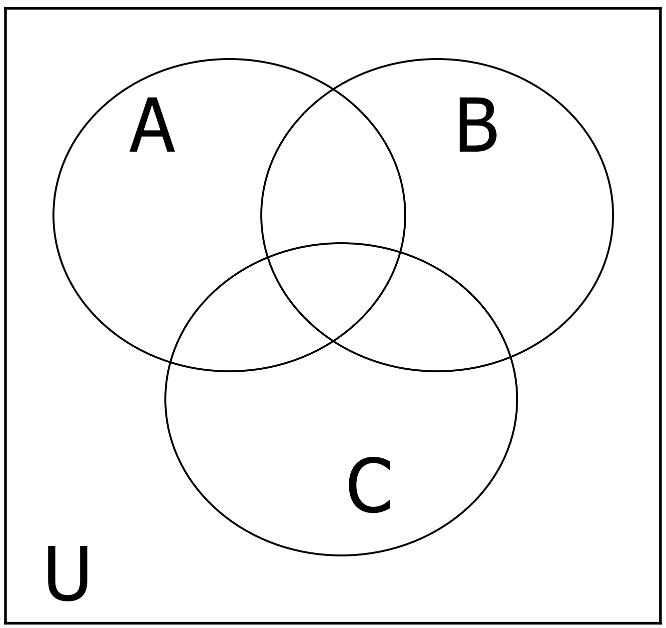
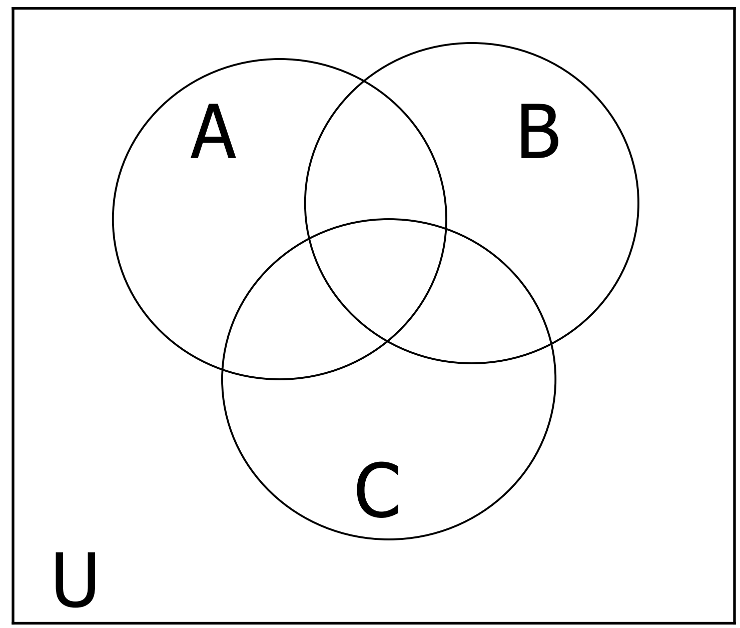


What would these look like??

$A \cap B \cap C$



$A \cup B \cup C$



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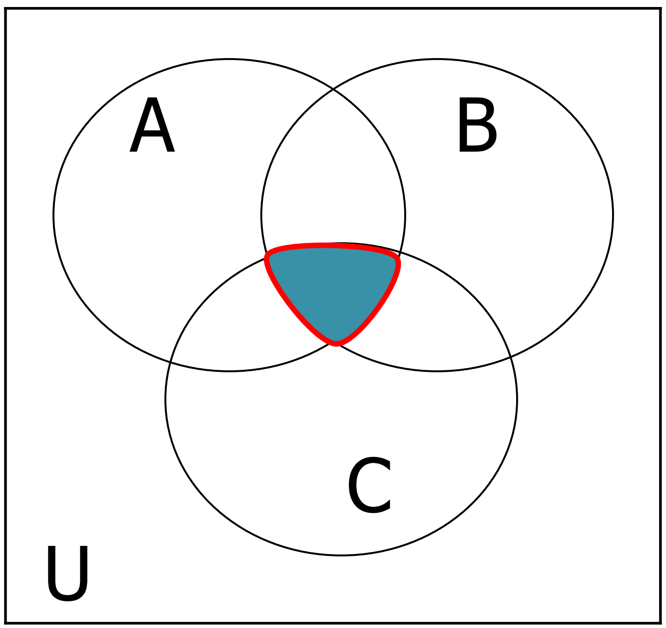
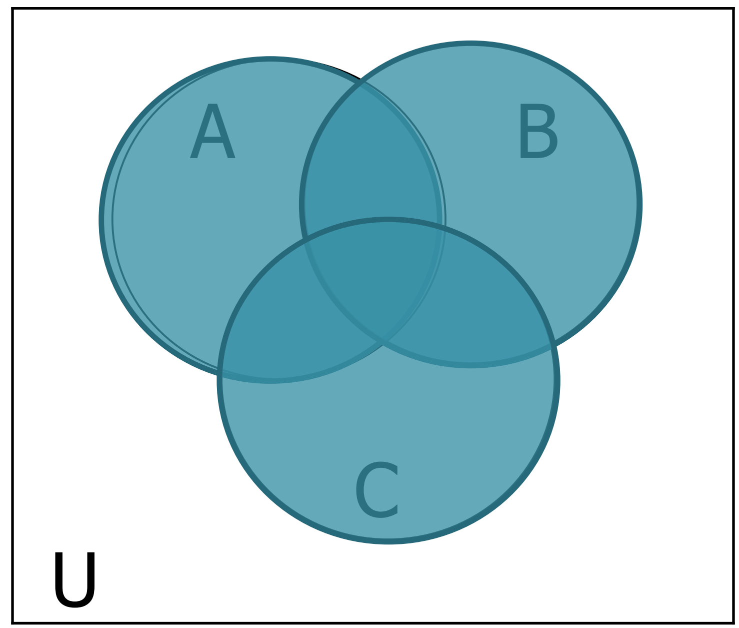


$A \cup B \cup C$



Consider the Universal Set consisting of the integers between 1 and 8. Given A={2,3,5,7}, B={2,4,6,8}, C={6} and D={3,4,5,6}, use Venn diagrams to help find the following.

$A \cap D$

$B \cup D$

A^C

$C^C \cup B$

$B^C \cap D$

Consider the Universal Set consisting of the integers between 1 and 8. Given A={2,3,5,7}, B={2,4,6,8}, C={6} and D={3,4,5,6}, use Venn diagrams to help find the following.

$A \cap D = \{3, 5\}$

$B \cup D = \{2, 3, 4, 5, 6, 8\}$

$A^C = \{1, 4, 6, 8\}$

$C^C \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$B^C \cap D = \{3, 5\}$

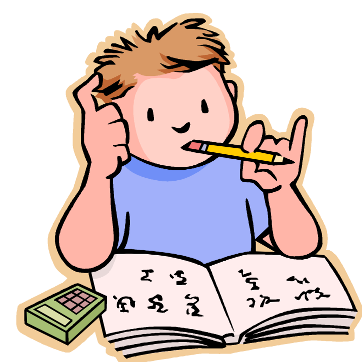
To Do: Tonight (Day 0)

1) Complete Day 0 HW Handout
2) Make sure you can access the website!
   ghhsicm.weebly.com
3) Get Syllabus & Honor Code Form
   signed by you AND your parent
4) Get Supplies for class (especially calculator!)

Our classroom could use:

Tissues!!! ☺

Dry Erase Markers



Homework Day 1 will be…

Packet p. 1-2

(you can start this on Day 0, if you want)