KEY
with simpler method
with colors :)

Unit 5 Day 7

Chain Rule

Warm Up ~

Find g(h(x)) and simplify.

1. $g(x) = \dfrac{2x-1}{3x^2+2}$; and $h(x) = x - 5$

Given h(x) = f(g(x)) below, identify f(x) and g(x).

2. $h(x) = \sqrt{x^2 - 7}$

Find an equation of a tangent line to the given function.

3. $g(x) = \dfrac{2x-1}{3x^2+2}; x = -2$

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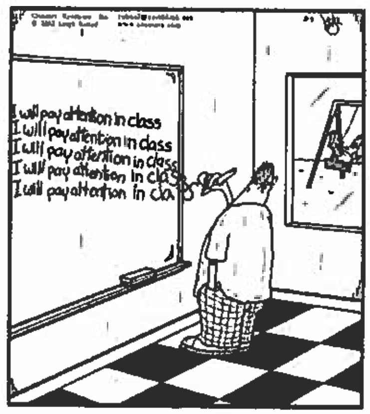

1) $g(h(x)) = g(x-5) = \dfrac{2(x-5)-1}{3(x-5)^2+2} = \dfrac{2x-10-1}{3x^2-30x+75+2} = \boxed{\dfrac{2x-11}{3x^2-30x+77}}$

$3(x^2-10x+25)+2$

$3x^2-30x+75+2$

2) $h(x)=\sqrt{x^2-7}$ + $h(x) = f(g(x))$

inside $= \boxed{g(x) = x^2-7}$

overall outer $= \boxed{f(x) = \sqrt{x}}$

3) $y' = \dfrac{\overset{\text{Lo d Hi}}{(3x^2+2)(2)} - \overset{\text{Hi d Lo}}{(2x-1)(6x)}}{\underset{\text{Lo}\cdot\text{Lo}}{(3x^2+2)(3x^2+2)}} = \dfrac{6x^2+4-\overset{-12x^2+6x}{(12x^2-6x)}}{9x^4+12x^2+4} = y' = \dfrac{-6x^2+6x+4}{9x^4+12x^2+4}$

$m = y'(-2) = \dfrac{-6(-2)^2+6(-2)+4}{9(-2)^4+12(-2)^2+4} = \dfrac{-24-12+4}{144+48+4} = \dfrac{-32}{196} = \dfrac{-8}{49} = m$

$y_1 = g(-2) = \dfrac{2(-2)-1}{3(-2)^2+2} = \dfrac{-4-1}{12+2} = \dfrac{-5}{14}$ Point$(-2, -5/14)$

$\boxed{y + 5/14 = -8/49(x+2)}$ Point-Slope OR $\boxed{y = \dfrac{-8}{49}x - \dfrac{67}{98}}$ Slope-Intercept

Warm Up ~ ANSWERS

Find g(h(x)) and simplify.

1. $g(x)=\dfrac{2x-1}{3x^2+2};\ and\ h(x)=x-5$

$$g(h(x))=g(x-5)$$

$$=\frac{2(x-5)-1}{3(x-5)^2+2}=\frac{2x-10-1}{3(x^2-10x+25)+2}$$

$$=\frac{2x-11}{3x^2-30x+77}$$

Given h(x) = f(g(x)) below, identify f(x) and g(x).

2. $h(x)=\sqrt{x^2-7} \quad f(x)=\sqrt{x} \quad g(x)=x^2-7$

Warm Up ~ ANSWERS

Find an equation of a tangent line to the given function.

3. $g(x)=\dfrac{2x-1}{3x^2+2};x=-2$

$$y+\frac{5}{14}=-\frac{8}{49}(x+2)$$

$$y=-\frac{8}{49}x-\frac{67}{98}$$

Notes Day 7 Chain Rule

Comparing Prior Skills to New Skills

Prior Skill

$\sqrt{x}\cdot(x^2+x)$ is a combination of two functions

Find $\dfrac{d}{dx}$ using Product Rule

New Skill

$\sqrt{(x^2+x)}$ is a composition of two functions

Find $\dfrac{d}{dx}$ using Chain Rule

The Chain Rule

Remember the composition of two functions?

$$f\circ g = f(g(x))$$

The chain rule is used to find the derivative of the composition of two functions.

$$\frac{d}{dx}[f(g(x))] = \underbrace{f'(g(x))}_{\text{d Outside with Inside left alone}}\cdot\underbrace{g'(x)}_{\text{d Inside part}}$$

(d Outside with Inside left alone) · (d Inside part)

d Outer · d Inner

x^5

$5x^4$

Chain Rule

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

$$y = (2x^3 + x + 7)^5 = (2x^3 + x + 7)^5$$

$+x'$

$1x$

$$\underbrace{5(2x^3+x+7)^4}_{\text{Derivative of outside function evaluated at inside function}} \cdot \underbrace{(6x^2+1)}_{\text{Derivative of inside function}}$$

$$5(6x^2+1)(2x^3+x+7)^4$$

$$\boxed{(30x^2+5)(2x^3+x+7)^4}$$

Chain Rule

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

$$y = (2x^3 + x + 7)^5$$

$$\frac{dy}{dx} = \underbrace{(5(2x^3 + x + 7)^4)}_{\text{Derivative of outside function evaluated at inside function}} \bullet \underbrace{(6x^2 + 1)}_{\text{Derivative of inside function}}$$

Ex: Find y' for $y=(x^2+1)^3$

$$\frac{d}{dx}[f(g(x))]=f'(g(x))g'(x)$$

$f(x) = x^3$

$g(x) = x^2 + 1$

$y=(x^2+1)^3$

$y' = \underbrace{3(x^2+1)^2}_{d\text{ Outer}} \cdot \underbrace{2x}_{d\text{ Inside}} = \boxed{6x(x^2+1)^2}$

Ex: Find y' for $y = (3x - 2x^2)^3$

$f(x) = x^3$

$g(x) = 3x - 2x^2$

$y=(3x-2x^2)^3$

$y' = 3(3x-2x^2)^2 \cdot \underbrace{(3-4x)}_{d\text{ Inside}}$

$3(3-4x)(3x-2x^2)^2$

$\boxed{(9-12x)(3x-2x^2)^2}$

Ex: Find y' for $y=(x^2+1)^3$

$$\frac{d}{dx}[f(g(x))]=f'(g(x))g'(x)$$

$f(x) = x^3$

$g(x) = x^2 + 1$

$y' = 3(x^2+1)^2(2x) = 6x(x^2+1)^2$

Ex: Find y' for $y = (3x - 2x^2)^3$

$f(x) = x^3$

$g(x) = 3x - 2x^2$

$y' = 3(3x - 2x^2)^2(3 - 4x)$

Examples: Find the derivative of the following.

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

Ex. $y = \sqrt{x+1}$ Rewrite as $(x+1)^{\frac{1}{2}}$

You Try!

Ex. $y = \sqrt{9x+1}$ Rewrite as $(9x+1)^{\frac{1}{2}}$

Examples ANSWERS: Find the derivative of the following.

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

Ex. $y = \sqrt{x+1}$ Rewrite as $(x+1)^{\frac{1}{2}}$

$$y' = \frac{1}{2\sqrt{x+1}}$$

You Try!

Ex. $y = \sqrt{9x+1}$ Rewrite as $(9x+1)^{\frac{1}{2}}$

$y' = \frac{1}{2}(9x+1)^{-1/2} \cdot 9$

$$y' = \frac{9}{2\sqrt{9x+1}}$$

$\frac{9}{2}(9x+1)^{-1/2}$

$= \frac{9}{2\sqrt{9x+1}}$

$(9x+1)^{1/2}$

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

Ex:
Find f'(x) for $f(x) = \sqrt[3]{(x^2+2)^2}$

$=$

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

Ex:
Find f'(x) for $f(x) = \sqrt[3]{(x^2+2)^2}$

$$= \left(x^2+2\right)^{2/3}$$

$$f'(x) = \frac{2}{3}\left(x^2+2\right)^{-1/3}(2x) = \frac{4x}{3\sqrt[3]{x^2+2}}$$

Differentiate

$g(t) = \frac{-7}{(2t-3)^2}$ rewritten as $g(t) = -7(2t-3)^{-2}$

$g(t) = -7(2t-3)^{-2}$

$g'(t) = \underline{14}(2t-3)^{-3} \cdot \underline{2}$

$g'(t) = 28(2t-3)^{-3}$

$g'(t) = \frac{28}{(2t-3)^3}$

Differentiate ANSWERS

$g(t) = \frac{-7}{(2t-3)^2}$ rewritten as $= -7(2t-3)^{-2}$

$g'(t) = 14(2t-3)^{-3}(2) = \frac{28}{(2t-3)^3}$

mult.

Differentiate- This needs the product and chain rule!

function

$h(x) = x^2 \cdot \sqrt{1-x^2}$ rewritten as

inside

inside

Remember the Product Rule?? $y = h(x) = x^2(1-x^2)^{1/2}$

$$\frac{d}{dx}[f(x)g(x)] = g(x)f'(x) + f(x)g'(x)$$

$f \cdot g$

$f = x^2$ $f' = 2x$

$g = (1-x^2)^{1/2}$ $g' =$ chain rule

$h'(x) = \overset{f' \cdot g}{(2x)(1-x^2)^{1/2}} + (x^2)(\text{chain rule})$

$h'(x) = 2x\sqrt{1-x^2} + \frac{x^2}{1}\left(\frac{-x}{\sqrt{1-x^2}}\right)$

$g = (1-x^2)^{1/2}$

$g = (1-x^2)^{1/2}$

$g' = \frac{1}{2}(1-x^2)^{-1/2} \cdot 2$

$g' = -x(1-x^2)$

$g' = \frac{-x}{\sqrt{1-x^2}}$

$(1-x^2)^{1/2}$

Differentiate- This needs the product and chain rule!

$h(x) = x^2\sqrt{1-x^2}$ rewritten as $x^2(1-x^2)^{1/2}$

Remember the Product Rule?? $2x\sqrt{1-x^2} - \frac{x^3}{\sqrt{1-x^2}}$

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$$

$f(x) = x^2$ $g(x) = (1-x^2)^{\frac{1}{2}}$

$$h'(x) = x^2\left(\frac{1}{\cancel{2}}\right)(1-x^2)^{-1/2}(-\cancel{2}x) + (1-x^2)^{1/2}(2x)$$

$$\boxed{h'(x) = -x^3(1-x^2)^{-1/2} + 2x(1-x^2)^{1/2}}$$

$f=(-x^4-1)^{1/2}$

$f'=\frac{1}{2}(-x^4-1)^{-1/2}\cdot -4x^3$

$f'=\frac{1}{2}(-4x^3)(-x^4-1)^{-1/2}$

$f'=-2x^3(-x^4-1)^{-1/2}$

$f'=\frac{-2x^3}{\sqrt{-x^4-1}}$

$(-x^4-1)^{1/2}$

Find the derivative of the following.

1. $y=\sqrt{-x^4-1}(-x-2)$

$f \cdot g$

$f=(-x^4-1)^{1/2}$ $\quad f'=$ chain rule

$g=-x-2$ $\quad g'=-1$

$y'=(\text{chain rule})(-x-2)+-1(-x^4-1)^{1/2}$

$y'=\left(\frac{-2x^3}{\sqrt{-x^4-1}}\right)(-x-2)-\sqrt{-x^4-1}$

$\boxed{\frac{2x^4+4x^3}{\sqrt{-x^4-1}}-\sqrt{-x^4-1}}$

ANSWER: Find the derivative of the following.

1. $y=\sqrt{-x^4-1}(-x-2)$

$$y'=-\sqrt{-x^4-1}+\frac{2x^4+4x^3}{\sqrt{-x^4-1}}$$

You Try! Find the derivative of the following.

2. $y = \overset{f}{(3x-1)} \cdot \overset{g}{(-3x^2-4)^{-3}}$

$f = 3x - 1$ $f' = 3$

$g = (-3x^2-4)^{-3}$

$g' = $ chain rule needed

$g' = -3(-3x^2-4)^{-4} \cdot (-6x)$

$g' = 18x(-3x^2-4)^{-4}$

$g' = \frac{18x}{(-3x^2-4)^4}$

$f' \cdot g + f \cdot g'$

$y' = 3(-3x^2-4)^{-3} + (3x-1) \cdot \frac{18x}{(-3x^2-4)^4}$

$y' = \frac{3}{(-3x^2-4)^3} + \frac{54x^2 - 18x}{(-3x^2-4)^4}$

You Try! ANSWER: Find the derivative of the following.

2. $y = (3x-1)(-3x^2-4)^{-3}$

Example Problems (GETTING your ANSWER to MATCH a GIVEN ANSWER)—Find the derivative of the following.

1. $y = x^3(2x-5)^4$

Example Problems (GETTING your ANSWER to MATCH a GIVEN ANSWER)—Find the derivative of the following.

1. $y = x^3(2x-5)^4$

$$\frac{dy}{dx} = (2x-5)^3(14x^3 - 15x^2)$$

Differentiate

$$f(x) = \frac{x}{\sqrt[3]{x^2+4}}$$ rewritten as

Quotient Rule $\dfrac{\text{Bot * Top' – Top * Bot'}}{(\text{Bot})^2}$

Differentiate

$$f(x) = \frac{x}{\sqrt[3]{x^2+4}} \text{ rewritten as } = \frac{x}{\left(x^2+4\right)^{1/3}}$$

Quotient Rule $\dfrac{\text{Bot * Top' – Top * Bot'}}{(\text{Bot})^2}$

$$f'(x) = \frac{(x^2+4)^{1/3}(1) - x(1/3)(x^2+4)^{-2/3}(2x)}{(x^2+4)^{2/3}}$$

$$f'(x) = \frac{(x^2+4)^{1/3} - (2x^2/3)(x^2+4)^{-2/3}}{(x^2+4)^{2/3}}$$

Differentiate

$$y=\left(\frac{3x-1}{x^2+3}\right)^2$$

$$y=\left(\frac{3x-1}{x^2+3}\right)^2$$

$$y'=2\left(\frac{3x-1}{x^2+3}\right)\cdot \text{d Inside}$$

Hi = 3x − 1 dHi = 3

Lo = x² + 3 dLo = 2x

$$(6x^2-2x)$$

$$\text{dInside} = \frac{(x^2+3)(3)-(3x-1)(2x)}{(x^2+3)(x^2+3)}$$

$$\text{dInner} = \frac{3x^2+9-6x^2+2x}{(x^2+3)^2} = \frac{-3x^2+2x+9}{(x^2+3)^2}$$

$$y'=2\left(\frac{3x-1}{x^2+3}\right)\left(\frac{-3x^2+2x+9}{(x^2+3)^2}\right)$$

$$=2\left(\frac{(3x-1)(-3x^2+2x+9)}{(x^2+3)^3}\right)$$

Differentiate

$$y=\left(\frac{3x-1}{x^2+3}\right)^2$$

$$y'=2\left(\frac{3x-1}{x^2+3}\right)^1\left(\frac{(x^2+3)(3)-(3x-1)(2x)}{(x^2+3)^2}\right)$$

$$=\frac{2(3x-1)(3x^2+9-6x^2+2x)}{(x^2+3)^3}$$

$$=\frac{2(3x-1)(-3x^2+2x+9)}{(x^2+3)^3}$$